On the quantization of massive 3-forms

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Abstract

Massive 3-forms are analyzed from the point of view of the Hamiltonian quantization using the gauge-unfixing approach and respectively the Batalin-Fradkin method. Both methods finally output a manifestly Lorentz covariant path-integral.

1. Introduction

The purpose of this paper is to present the problem of the Hamiltonian quantization of the massive 3-form in the framework of the gauge-unfixing (GU) approach [1]–[2] and respectively of the Batalin-Fradkin (BF) method [3]–[5] based on path integral. The main idea is to associate with the original second-class theory an equivalent first-class system. The associated first-class system has to satisfy the following requirements: its number of physical degrees of freedom coincides with that of the original second-class theory, the algebras of classical observables are isomorphic and the first-class Hamiltonian restricted to the original constraint surface reduces to the original canonical Hamiltonian. The above isomorphism renders the equivalence of the two systems also at the level of the path integral quantization and hence allows the replacement of the Hamiltonian path integral for the original second-class theory with that of the equivalent first-class system.

2. Gauge unfixing method

The starting point is a bosonic dynamic system with the phase-space locally parameterized by n canonical pairs $z^a = \left(q^i,\ p_i\right)$, endowed with the canonical Hamiltonian H_c, and subject to the purely second-class constraints

$$\chi_{\alpha_0}\left(z^a\right) \approx 0, \qquad \alpha_0 = \overline{1, 2M_0}, \tag{1}$$

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Assume that one can split the second-class constraint set (1) into two subsets

$$\chi_{\alpha_0}(z^a) \equiv \left(G_{\bar{\alpha}_0}(z^a), C^{\bar{\beta}_0}(z^a)\right) \approx 0, \qquad \bar{\alpha}_0, \bar{\beta}_0 = \overline{1, M_0}. \tag{2}$$

such that

$$\left[G_{\bar{\alpha}_0}, G_{\bar{\beta}_0}\right] = D^{\bar{\gamma}_0}_{\bar{\alpha}_0\bar{\beta}_0} G_{\bar{\gamma}_0}. \tag{3}$$

Relations (3) yield the subset

$$G_{\bar{\alpha}_0}(z^a) \approx 0 \tag{4}$$

to be first-class. The second-class behaviour of the overall constraint set ensures that $C^{\bar{\alpha}_0}(z^a) \approx 0$ may be regarded as some gauge-fixing conditions for this first-class set.

We introduce an operator $\hat{X}$ [6]–[8] that associates with every smooth function F on the original phase-space an application $\hat{X}F$

$$\hat{X}F = F - C^{\bar{\alpha}_0}\left[G_{\bar{\alpha}_0}, F\right] + \tfrac{1}{2} C^{\bar{\alpha}_0} C^{\bar{\beta}_0}\left[G_{\bar{\alpha}_0}, \left[G_{\bar{\beta}_0}, F\right]\right] - \cdots, \tag{5}$$

which is in strong involution with the functions $G_{\bar{\alpha}_0}$.

The original second-class theory and respectively the gauge-unfixed system are classically equivalent since they possess the same number of physical degrees of freedom $\mathcal{N}_O = \frac{1}{2}(2n - 2M_0) = \mathcal{N}_{GU}$ and the corresponding algebras of classical observables are isomorphic. Consequently, the two systems become also equivalent at the level of the path integral quantization and we can to replace the the Hamiltonian path integral of the original second-class theory with the Hamiltonian path integral of the gauge-unfixed first-class system.

In the sequel we shall quantize the massive 3-forms on behalf of GU method. We start from the Lagrangian action of massive 3-forms in $D \geq 4$ [9]–[10]

$$S_0^L\left[A_{\mu\nu\rho}\right] = \int d^D x \left(-\tfrac{1}{48} F_{\mu\nu\rho\lambda} F^{\mu\nu\rho\lambda} - \tfrac{m^2}{12} A_{\mu\nu\rho} A^{\mu\nu\rho}\right). \tag{6}$$

By performing the canonical analysis [11]–[12] of this model, there result the irreducible second-class constraints

$$\chi^{(1)ij} \equiv \pi^{0ij} \approx 0, \tag{7}$$

$$\chi^{(2)}_{ij} \equiv 3\partial^k \pi_{kij} - \tfrac{m^2}{2} A_{0ij} \approx 0, \tag{8}$$

along with the canonical Hamiltonian

$$H_c(x^0) = \int d^{D-1}x \left(-3\pi_{ijk}\pi^{ijk} + \tfrac{1}{48} F_{ijk} F^{ijk} + \tfrac{m^2}{12} A_{\mu\nu\rho} A^{\mu\nu\rho} - 3A_{0ij}\partial_k \pi^{kij}\right). \tag{9}$$

According to the GU method we consider (8) as the first-class constraint set and the remaining constraints (7) as the corresponding canonical gauge conditions and redefine the first-class constraints as

$$G^{ij} \equiv -\tfrac{1}{m^2}\left(3\partial_k \pi^{kij} - \tfrac{m^2}{2}A^{0ij}\right) \approx 0. \qquad (10)$$

The first-class Hamiltonian with respect to (10) follows from relation (5)

$$\hat{X}H_c(y^0) = H_c(y^0) - \int d^{D-1}y\left[\tfrac{1}{2}\pi_{0ij}\partial_k A^{kij} - \tfrac{1}{12m^2}\partial_{[i}\pi_{jk]0}\partial^{[i}\pi^{jk]0}\right]. \qquad (11)$$

An irreducible set of constraints can always be replaced by a reducible one by introducing constraints that are consequences of the ones already at hand [13]. In view of this, we supplement (10) with one more constraint, $G^i \equiv -\frac{m^2}{2}\partial_j G^{ji} \approx 0$, such that the new constraint set

$$G^{ij} \equiv -\tfrac{1}{m^2}\left(3\partial_k \pi^{kij} - \tfrac{m^2}{2}A^{0ij}\right) \approx 0, \qquad G^i \equiv -\tfrac{m^2}{2}\partial_j A^{0ji} \approx 0, \qquad (12)$$

remains first-class and, moreover, becomes off-shell second-order reducible. If we make the transformations $A_{0ij} \to -\frac{1}{m^2}\Pi_{ij}$ and $\pi^{0ij} \to m^2 B^{ij}$, then the constraints (12) become

$$G^{ij} \equiv -\tfrac{1}{m^2}\left(3\partial_j \pi^{ji} + \tfrac{1}{2}\Pi^{ij}\right) \approx 0, \qquad G^i \equiv \tfrac{1}{2}\partial_j \Pi^{ji} \approx 0, \qquad (13)$$

while the first-class Hamiltonian (11) takes the form

$$H_{GU}(y^0) = \int d^{D-1}y\left[\tfrac{1}{48}F_{ijkl}F^{ijkl} + \tfrac{m^2}{12}\left(\partial_{[i}B_{jk]} + A_{ijk}\right)\left(\partial^{[i}B^{jk]} + A^{ijk}\right)\right.$$
$$\left. - 3\pi_{ijk}\pi^{ijk} - \tfrac{1}{4m^2}\Pi_{ij}\Pi^{ij} + \tfrac{1}{m^2}\Pi_{ij}\left(3\partial_k \pi^{kij} + \tfrac{1}{2}\Pi^{ij}\right)\right]. \qquad (14)$$

Due to the equivalence between the reducible first-class system and the original second-class theory, one can replace the Hamiltonian path integral of massive 3-forms with that associated with the reducible first-class system. The argument of the exponential from the Hamiltonian path integral of the second-order reducible first-class system reads as

$$S_{GU} = \int d^D x\left(\pi_{ijk}\dot{A}^{ijk} + \Pi_{ij}\dot{B}^{ij} - \mathcal{H}_{GU} - \lambda_{ij}G^{ij} - \lambda_i G^i\right). \qquad (15)$$

We enlarge the original phase-space with the Lagrange multipliers $\left\{\bar{\lambda}_{ij}, \lambda_i\right\}$ and with their canonical momenta $\left\{p^{ij}, p^i\right\}$ and in order to preserve the number of physical degree of freedom we add the first-class constraints

$$p^{ij} \approx 0, \qquad p^i \approx 0. \qquad (16)$$

If we perform the transformations $\Pi^{ij} \to \Pi^{ij}$ and $\lambda_{ij} \to \bar{\lambda}_{ij} = \lambda_{ij} - \Pi_{ij}$ in the path integral, then the argument of the exponential from the Hamiltonian path integral for the theory with the phase-space locally parameterized by fields/momenta $\left\{A_{ijk}, B_{ij}, \bar{\lambda}_{ij}, \lambda_i, \pi^{ijk}, \Pi^{ij}, p^{ij}, p^i\right\}$ and subject to the first-class constraints (13) and (16) reads as

$$S'_{GU} = \int d^D x \left[\pi_{ijk}\dot{A}^{ijk} + \Pi_{ij}\dot{B}^{ij} + p^{ij}\dot{\bar{\lambda}}_{ij} + p^i\dot{\lambda}_i - \tfrac{1}{48}F_{ijkl}F^{ijkl} \right.$$
$$- \tfrac{m^2}{12}\left(\partial_{[i}B_{jk]} + A_{ijk}\right)\left(\partial^{[i}B^{jk]} + A^{ijk}\right) + 3\pi_{ijk}\pi^{ijk} + \tfrac{1}{4m^2}\Pi_{ij}\Pi^{ij}$$
$$\left. + \tfrac{1}{m^2}\bar{\lambda}_{ij}\left(3\partial_k\pi^{kij} + \tfrac{1}{2}\Pi^{ij}\right) - \tfrac{1}{2}\lambda_j\left(\partial_j\Pi^{ji}\right) - \Lambda_{ij}p^{ij} - \Lambda_i p^i\right]. \tag{17}$$

Performing in (17) the integration over $\left\{\pi^{ijk}, \Pi^{ij}, p^{ij}, p^i, \Lambda_{ij}, \Lambda_i\right\}$ and making the notations $\frac{1}{m^2}\bar{\lambda}_{ij} \equiv \bar{A}_{ij0}$ and $\frac{1}{4}\lambda_i \equiv B_{i0}$, the functional (17) associated with the reducible first-class system takes now a manifestly Lorentz covariant form

$$\tilde{S}_{GU}\left[\bar{B}_{\mu\nu}, \bar{A}_{\mu\nu\rho}\right] = \int d^D x \left[-\tfrac{1}{48}\bar{F}_{\mu\nu\rho\lambda}\bar{F}^{\mu\nu\rho\lambda} \right.$$
$$\left. - \tfrac{1}{12}\left(F_{\mu\nu\rho} - m\bar{A}_{\mu\nu\rho}\right)\left(F^{\mu\nu\rho} - m\bar{A}^{\mu\nu\rho}\right)\right], \tag{18}$$

with

$$\bar{A}_{\mu\nu\rho} \equiv \left(\bar{A}_{0ij}, A_{ijk}\right), \qquad \bar{F}_{\mu\nu\rho\lambda} = \partial_{[\mu}\bar{A}_{\nu\rho\lambda]}, \tag{19}$$

$$\bar{B}_{\mu\nu} = -\tfrac{1}{m}B_{\mu\nu}, \qquad F_{\mu\nu\rho} = \partial_{[\mu}\bar{B}_{\nu\rho]}, \tag{20}$$

and describes precisely the (Lagrangian) Stückelberg coupling [14] between the 2-form $\bar{B}_{\mu\nu}$ and 3-form $\bar{A}_{\mu\nu\rho}$.

In the sequel we show how the massive 3-form gets related to the 4-form gauge fields. In order to do this we start from the GU system constructed in the above (subject to the first-class constraints (10) whose evolution is governed by the first-class Hamiltonian (11)) and consider the quantities

$$\mathcal{F}_{ijk} = A_{ijk} + \tfrac{1}{m^2}\partial_{[i}\pi_{jk]0}, \qquad \mathcal{F}_{0ij} = A_{0ij}, \tag{21}$$

which are in involution with first-class constraints (10). We define

$$\mathcal{W}_{\mu\nu\rho\lambda} = \partial_{[\mu}\mathcal{F}_{\nu\rho\lambda]}, \quad \textit{where} \quad \mathcal{F}_{\mu\nu\rho} \equiv \left\{\mathcal{F}_{0ij}, \mathcal{F}_{ijk}\right\}. \tag{22}$$

By direct computation, it follows that

$$\partial^\mu\mathcal{W}_{\mu\nu\rho\lambda} = m^2\mathcal{F}_{\nu\rho\lambda} + \mathcal{O}\left(G^{ij}\right). \tag{23}$$

From (23) we obtain that

$$\partial^{\nu}\mathcal{F}_{\nu\rho\lambda}=0. \tag{24}$$

The fields $\mathcal{F}_{\mu\nu\rho}$ can be written in terms of a 4-form $B^{\alpha\beta\gamma\delta}$

$$\mathcal{F}_{\mu\nu\rho}=\tfrac{1}{5!}\,\varepsilon_{\mu\nu\rho\alpha\beta\gamma\delta\epsilon}F^{\alpha\beta\gamma\delta\epsilon}, \tag{25}$$

where $F^{\alpha\beta\gamma\delta\epsilon}=\partial^{[\alpha}B^{\beta\gamma\delta\epsilon]}$. Consequently, we enlarge the phase-space by adding the bosonic fields/momenta $\left(B^{\alpha\beta\gamma\delta},\pi_{\alpha\beta\gamma\delta}\right)$. When we replace (25) in (10) the constraints set takes the form

$$G^{ij}\equiv-\tfrac{1}{m^2}\left(3\partial_k\pi^{kij}-\tfrac{m^2}{2\cdot5!}\varepsilon^{0ijklnqr}F_{klnqr}\right)\approx0, \tag{26}$$

remains first-class and becomes second-order reducible. In order to preserve the number of physical degrees of freedom we must to impose the constraints

$$G^{(1)ijk}\equiv4\partial_l\pi^{lijk}\approx0,\qquad G^{(2)ijk}\equiv\pi^{0ijk}\approx0. \tag{27}$$

The constraints (26) and (27) are first-class and reducible. The first-class Hamiltonian becomes

$$\begin{aligned}H'_{GU}(y^0)=\int d^7y\,\Big[&\tfrac{1}{48}F_{ijkl}F^{ijkl}+\tfrac{m^2}{12}A_{ijk}A^{ijk}+\tfrac{1}{3!}\varepsilon_{0ijklnqr}A^{ijk}\pi^{lnqr}\\&+\tfrac{m^2}{2\cdot5!}F_{ijkln}F^{ijkln}-3\pi_{ijk}\pi^{ijk}+\tfrac{12}{m^2}\pi_{ijkl}\pi^{ijkl}\\&+\tfrac{1}{5!}\,\varepsilon_{0ijlnqrs}F^{lnqrs}\left(3\partial_k\pi^{kij}-\tfrac{m^2}{2\cdot5!}\varepsilon^{0ijl'n'q'r's'}F_{l'n'q'r's'}\right)\Big].\end{aligned} \tag{28}$$

The argument of the exponential from the Hamiltonian path integral of the above reducible first-class system as

$$\begin{aligned}S'_{GU}=\int d^8x\,\Big[&\pi_{ijk}\dot{A}^{ijk}+\pi_{0ijk}\dot{B}^{0ijk}+\pi_{ijkl}\dot{B}^{ijkl}-\mathcal{H}'_{GU}\\&-\lambda_{ij}G^{ij}-\lambda^{(1)}_{ijk}G^{(1)ijk}-\lambda^{(2)}_{ijk}G^{(2)ijk}\Big].\end{aligned} \tag{29}$$

After integrating out the auxiliary fields and performing some field redefinitions, we obtain

$$\begin{aligned}\tilde{S}_{GU}\left[A_{\mu\nu\rho},B_{\alpha\beta\gamma\delta}\right]=\int d^8x\,\Big[&-\tfrac{1}{48}F_{\mu\nu\rho\lambda}F^{\mu\nu\rho\lambda}-\tfrac{m^2}{3!\cdot5!}\varepsilon_{\mu\nu\rho\alpha\beta\gamma\delta\epsilon}A^{\mu\nu\rho}F^{\alpha\beta\gamma\delta\epsilon}\\&+\tfrac{m^2}{2\cdot5!}\,F_{\alpha\beta\gamma\delta\epsilon}F^{\alpha\beta\gamma\delta\epsilon}\Big],\end{aligned} \tag{30}$$

and describes a generalized Chern-Simons coupling [15]-[17] between the 3-form $A_{\mu\nu\rho}$ and 4-form $B_{\alpha\beta\gamma\delta}$.

3. BF method

In order to construct a first-class system equivalent to the starting second-class one (subject to the second-class constraints (1)) in the framework of the BF approach we enlarge the original phase-space with $(\zeta^{\alpha})_{\alpha=\overline{1,2M}}$, $(M \geq M_0)$. The next step is to construct a set of independent, smooth, real functions defined on the extended phase-space, $(G_A(z,\zeta))_{A=\overline{1,M_0+M}}$ such that

$$G_{\alpha_0}(z,0) \equiv \chi_{\alpha_0}(z), \quad G_{\bar{A}}(z,0) \equiv 0, \quad [G_A, G_B] = 0, \tag{31}$$

where $\bar{A} = \overline{2M_0+1, M_0+M}$. In the last step we generate a smooth, real function, defined on the extended phase-space, $H_{BF}(z,\zeta)$ with the properties

$$H_{BF}(z,0) \equiv H_c(z), \qquad [H_{BF}, G_A] = V_A{}^{B} G_B. \tag{32}$$

The previous steps unravel a dynamic system subject to the first-class constraints $(G_A(z,\zeta))_{A=\overline{1,M_0+M}} \approx 0$ whose evolution is governed by the first-class Hamiltonian $H_{BF}(z,\zeta)$. If we denote by $\mathcal{S}_{BF}$ the BF system, then $\mathcal{S}_{BF}$ is classically equivalent with $\mathcal{S}_O$, since both of them display the same number of physical degrees of freedom

$$\mathcal{N}_O = \frac{1}{2}(2n - 2M_0) = \frac{1}{2}[2(n+M) - 2(M_0+M)] = \mathcal{N}_{BF}, \tag{33}$$

the corresponding algebras of classical observables are isomorphic. Consequently, $\mathcal{S}_{BF}$ and $\mathcal{S}_O$ become also equivalent at the level of the path integral quantization and we can to replace the the Hamiltonian path integral of the original second-class theory with that of the BF first-class system.

In the case of the massive 3-forms we enlarge the original phase-space by adding the bosonic fields/momenta $(B^{\mu\nu}, \Pi_{\mu\nu})_{\mu,\nu=\overline{0,D-1}}$. The constraints $G_A(z,\zeta) \approx 0$ gain in this case the concrete form

$$G^{(1)ij} \equiv \chi^{(1)ij} + mB^{ij} \approx 0, \quad G^{(2)}_{ij} \equiv \chi^{(2)}_{ij} - \frac{m}{2}\Pi_{ij} \approx 0, \quad G \equiv \Pi_{0i} \approx 0. \tag{34}$$

It is easy to check that they form an irreducible first-class constraint set. The first-class Hamiltonian complying with the general requirements (32) is expressed by

$$\begin{aligned} H_{BF}(x^0) = H_c(x^0) + \int d^{D-1}x \Big[& \tfrac{1}{4}\Pi^{ij}\Pi_{ij} - \tfrac{1}{m}\Pi^{ij}\left(3\partial^k \pi_{kij} - \frac{m^2}{2}A_{0ij}\right) \\ & - \tfrac{1}{3}\left(mA^{ijk} - \partial^{[i}B^{jk]}\right)\partial_{[i}B_{jk]} - \tfrac{1}{4}\partial^{[i}B^{j]0}\left(mA_{0ij} + \Pi_{ij}\right)\Big]. \end{aligned} \tag{35}$$

The argument of the exponential from the Hamiltonian path integral of the above BF first-class system, equivalent with that of massive 3-forms takes the form

$$S_{BF} = \int d^D x \left(\pi_{0ij} \dot{A}^{0ij} + \pi_{ijk} \dot{A}^{ijk} + \Pi_{0i} \dot{B}^{0i} + \Pi_{ij} \dot{B}^{ij} - \mathcal{H}_{BF} \right.$$
$$\left. - \lambda^{(1)ij} G^{(1)}_{ij} - \lambda^{(2)ij} G^{(2)}_{ij} - \lambda^i G_i \right). \tag{36}$$

Integrating in the path integral over $\{\pi_{ijk}\}$ and employing the change of variables $\Pi_{ij} \rightarrow \Pi'_{ij} = \Pi_{ij} + mA_{0ij}$ and $\pi_{0ij} \rightarrow \pi'_{0ij} = \pi_{0ij} + mB_{ij}$, the argument of the exponential from the Hamiltonian path integral becomes

$$S'_{BF} = \int d^D x \left[\pi'_{0ij} \dot{A}^{0ij} + \Pi'_{ij} \dot{B}^{ij} + \Pi_{0i} \dot{B}^{0i} \right.$$
$$- \tfrac{1}{48} F_{ijkl} F^{ijkl} - \tfrac{1}{12} \left(2\partial_{[i} B_{jk]} - mA_{ijk} \right) \left(2\partial^{[i} B^{jk]} - mA^{ijk} \right)$$
$$- \tfrac{1}{12} \left(\partial_0 A_{ijk} - \tfrac{1}{m} \partial_{[i} \Pi'_{jk]} + \partial_{[i} \lambda^{(2)}_{jk]} \right) \left(\partial^0 A^{ijk} - \tfrac{1}{m} \partial^{[i} \Pi'^{jk]} + \partial^{[i} \lambda^{(2)jk]} \right)$$
$$\left. - \tfrac{1}{4} \Pi'_{ij} \Pi'^{ij} + \tfrac{1}{4} \partial^{[i} B^{j]0} \Pi'_{ij} - \lambda^{(1)ij} \pi'_{0ij} - m\lambda^{(2)ij} \Pi'_{ij} - \lambda^i \Pi_{0i} \right]. \tag{37}$$

Performing in the last form of the path integral the change of variables $\Pi'_{ij} \rightarrow \bar{A}_{0ij} = \frac{1}{m} \Pi'_{ij} - \lambda^{(2)}_{ij}$ and $\lambda^{(2)}_{ij} \rightarrow \lambda^{(2)}_{ij}$ the argument of the exponential from the path integral is turned into

$$S''_{BF} = \int d^D x \left[\pi'_{0ij} \dot{A}^{0ij} + m \left(\bar{A}_{0ij} + \lambda^{(2)}_{ij} \right) \dot{B}^{ij} + \Pi_{0i} \dot{B}^{0i} \right.$$
$$- \tfrac{1}{48} F_{ijkl} F^{ijkl} - \tfrac{1}{12} \left(\partial_0 A_{ijk} - \partial_{[i} \bar{A}_{jk]0} \right) \left(\partial^0 A^{ijk} - \partial^{[i} \bar{A}^{jk]0} \right)$$
$$+ \tfrac{m}{4} \partial^{[i} B^{j]0} \left(\bar{A}_{0ij} + \lambda^{(2)}_{ij} \right) - \tfrac{1}{12} \left(2\partial_{[i} B_{jk]} - mA_{ijk} \right) \left(2\partial^{[i} B^{jk]} - mA^{ijk} \right)$$
$$\left. - \tfrac{m^2}{4} \bar{A}_{0ij} \bar{A}^{0ij} + \tfrac{m^2}{4} \lambda^{(2)}_{ij} \lambda^{(2)ij} - \lambda^{(1)ij} \pi'_{0ij} - \lambda^i \Pi_{0i} \right]. \tag{38}$$

Integrating in the path integral over $\left\{ \pi'_{0ij}, \lambda^{(1)ij}, \Pi_{0i}, \lambda^i, A^{0ij}, \lambda^{(2)}_{ij} \right\}$ the argument of the exponential reduces to

$$S'''_{BF} = \int d^D x \left[-\tfrac{1}{2 \cdot 4!} F_{ijkl} F^{ijkl} - \tfrac{1}{2 \cdot 3!} \left(\partial_0 A_{ijk} - \partial_{[i} \bar{A}_{jk]0} \right) \left(\partial^0 A^{ijk} - \partial^{[i} \bar{A}^{jk]0} \right) \right.$$
$$- \tfrac{1}{4} \left[\left(\partial_0 \bar{B}_{ij} + \partial_{[i} \bar{B}_{j]0} \right) - m\bar{A}_{0ij} \right] \left[\left(\partial^0 \bar{B}^{ij} + \partial^{[i} \bar{B}^{j]0} \right) - m\bar{A}^{0ij} \right]$$
$$- \tfrac{1}{2 \cdot 3!} \left(\partial_{[i} \bar{B}_{jk]} - mA_{ijk} \right) \left(\partial^{[i} \bar{B}^{jk]} - mA^{ijk} \right). \tag{39}$$

where $\bar{B}_{ij} = 2B_{ij}$ and $\bar{B}_{0i} \equiv \frac{1}{2}B_{0i}$. The last functional associated with the equivalent first-class system takes now a manifestly Lorentz covariant form

$$\tilde{S}_{BF}\left[\bar{B}_{\mu\nu}, \bar{A}_{\mu\nu\rho}\right] = \int d^D x \left[-\frac{1}{48}\bar{F}_{\mu\nu\rho\lambda}\bar{F}^{\mu\nu\rho\lambda}\right. \tag{40}$$

$$\left. -\frac{1}{12}\left(F_{\mu\nu\rho} - m\bar{A}_{\mu\nu\rho}\right)\left(F^{\mu\nu\rho} - m\bar{A}^{\mu\nu\rho}\right)\right], \tag{41}$$

with

$$\bar{A}_{\mu\nu\rho} = \left(\bar{A}_{0ij}, A_{ijk}\right), \qquad \bar{F}_{\mu\nu\rho\lambda} = \partial_{[\mu}\bar{A}_{\nu\rho\lambda]}, \tag{42}$$

$$\bar{B}_{\mu\nu} = \left(\bar{B}_{0i}, \bar{B}_{ij}\right), \qquad F_{\mu\nu\rho} = \partial_{[\mu}\bar{B}_{\nu\rho]}, \tag{43}$$

and describes the (Lagrangian) Stückelberg coupling between the 2-form $\bar{B}_{\mu\nu}$ and the 3-form $\bar{A}_{\mu\nu\rho}$.

4. Conclusion

We analyzed the problem of the Hamiltonian quantization of the massive 3-forms using GU and respectively BF methods. In the framework of the first approach, starting from the original canonical Hamiltonian, we generated a first-class Hamiltonian with respect to the first-class constraint subset. We built the Hamiltonian path integral of the GU first-class system and after integrating out the auxiliary fields and performing some variable redefinitions the path integral finally takes a manifestly Lorentz covariant form. The second approach involves an appropriate extension of the original phase-space and then the construction of a first-class system on the extended phase-space that reduces to the original, second-class theory in the zero limit of all extra variables. The Hamiltonian path integral of the BF first-class system takes, after integrating out some of the variables and performing some field redefinitions, a manifestly Lorentz covariant form. Both approaches require appropriate extensions of the phase-space in order to render a manifestly covariant path integral.

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