T-duality and noncommutativity in type IIB superstring theory*

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Abstract

In this article we investigate noncommutativity of $D9$-brane world-volume embedded in space-time of type IIB superstring theory. On the solution of boundary conditions, the initial coordinates become noncommutative. We show that noncommutativity relations are connected by $N = 1$ supersymmetry transformations and noncommutativity parameters are components of $N = 1$ supermultiplet. In addition, we show that background fields of the effective theory (initial theory on the solution of boundary conditions) and noncommutativity parameters represent the background fields of the T-dual theory.

1. Introduction

Dp-branes with odd value of p are stable in type IIB superstring theory. As a particular choice, besides $D9$-brane, we can embed $D5$-brane in ten dimensional space-time of the type IIB theory in pure spinor formulation (up to the quadratic terms)[1, 2]. In the present paper we investigate the noncommutativity of $D9$-brane using canonical approach. The case of $D5$-brane has been considered in Ref.[3]. Also we investigate the supersymmetry of noncommutativity relations [4].

We choose Neumann boundary conditions for all bosonic coordinates x^μ. The boundary condition for fermionic coordinates, $(\theta^\alpha - \bar\theta^\alpha)|_0^\pi = 0$ produces additional one for their canonically conjugated momenta, $(\pi_\alpha - \bar\pi_\alpha)|_0^\pi = 0$. We treat boundary conditions as canonical constraints [5, 6, 7, 3]. Using their consistency conditions, we rewrite them in compact σ-dependent form. We find that they are of the second class and on these solutions, we obtain initial coordinates in terms of effective coordinates and momenta. Presence of the momenta is source of noncommutativity. Noncommutativity relations are consistent with $N = 1$ supersymmetry transformations.

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We obtain that noncommutativity parameters contain only odd powers of background fields antisymmetric under world-sheet parity transformation $\Omega : \sigma \to -\sigma$ and form one $N = 1$ supermultiplet. This result represents a supersymmetric generalization of the result obtained by Seiberg and Witten [8]. We also show that effective background fields and noncommutative parameters are the background fields of T-dual theory.

2. Type IIB superstring theory and canonical analysis

We will investigate pure spinor formulation [2, 9, 7, 3] of type IIB theory, neglecting ghost terms and keeping quadratic ones

$$S = \kappa \int_\Sigma d^2\xi \partial_+ x^\mu \Pi_{+\mu\nu} \partial_- x^\nu \qquad (1)$$
$$+ \int_\Sigma d^2\xi \left[-\pi_\alpha \partial_- (\theta^\alpha + \Psi^\alpha_\mu x^\mu) + \partial_+ (\bar\theta^\alpha + \bar\Psi^\alpha_\mu x^\mu) \bar\pi_\alpha + \frac{1}{2\kappa} \pi_\alpha F^{\alpha\beta} \bar\pi_\beta \right] ,$$

where $\Pi_{\pm\mu\nu} = B_{\mu\nu} \pm \frac{1}{2} G_{\mu\nu}$ and $\partial_\pm = \partial_\tau \pm \partial_\sigma$. All background fields are constant. The space-time coordinates are labeled by x^μ $(\mu = 0, 1, 2, \dots, 9)$ and the fermionic ones by same chirality Majorana-Weyl spinors θ^α and $\bar\theta^\alpha$. The variables π_α and $\bar\pi_\alpha$ are canonically conjugated to the coordinates θ^α and $\bar\theta^\alpha$, respectively.

Canonical Hamiltonian is of the form

$$H_c = \int d\sigma \mathcal{H}_c \, , \quad \mathcal{H}_c = T_- - T_+ \, , \quad T_\pm = t_\pm - \tau_\pm \, , \qquad (2)$$

where

$$t_\pm = \mp \frac{1}{4\kappa} G^{\mu\nu} I_{\pm\mu} I_{\pm\nu} \, , \quad I_{\pm\mu} = \pi_\mu + 2\kappa \Pi_{\pm\mu\nu} x'^\nu + \pi_\alpha \Psi^\alpha_\mu - \bar\Psi^\alpha_\mu \bar\pi_\alpha \, , \qquad (3)$$
$$\tau_+ = (\theta'^\alpha + \Psi^\alpha_\mu x'^\mu)\pi_\alpha - \frac{1}{4\kappa} \pi_\alpha F^{\alpha\beta} \bar\pi_\beta \, , \tau_- = (\bar\theta'^\alpha + \bar\Psi^\alpha_\mu x'^\mu)\bar\pi_\alpha + \frac{1}{4\kappa} \pi_\alpha F^{\alpha\beta} \bar\pi_\beta \, .$$

Varying Hamiltonian H_c, we obtain

$$\delta H_c = \delta H_c^{(R)} - [\gamma^{(0)}_\mu \delta x^\mu + \pi_\alpha \delta\theta^\alpha + \delta\bar\theta^\alpha \bar\pi_\alpha]|_0^\pi \, , \qquad (4)$$

where $\delta H_c^{(R)}$ is regular term without τ and σ derivatives of supercoordinates and supermomenta variations and

$$\gamma^{(0)}_\mu = \Pi_{+\mu}{}^\nu I_{-\nu} + \Pi_{-\mu}{}^\nu I_{+\nu} + \pi_\alpha \Psi^\alpha_\mu + \bar\Psi^\alpha_\mu \bar\pi_\alpha \, . \qquad (5)$$

As a time translation generator Hamiltonian has to be differentiable with respect to coordinates and their canonically conjugated momenta which produces

$$\left[\gamma^{(0)}_\mu \delta x^\mu + \pi_\alpha \delta\theta^\alpha + \delta\bar\theta^\alpha \bar\pi_\alpha \right] \Big|_0^\pi = 0 \, . \qquad (6)$$

Embedding $D9$-brane means that we impose Neumann boundary conditions for x^μ coordinates, $\gamma_\mu^{(0)}|_0^\pi = 0$. Fermionic boundary condition $(\theta^\alpha - \bar\theta^\alpha)\Big|_0^\pi = 0$ preserves half of the initial $N = 2$ supersymmetry and it produces additional boundary condition for fermionic momenta $(\pi_\alpha - \bar\pi_\alpha)|_0^\pi = 0$.

Using standard Poisson algebra, the consistency procedure for $\gamma_\mu^{(0)}$ produces an infinite set of constraints

$$\gamma_\mu^{(n)} \equiv \{H_c, \gamma_\mu^{(n-1)}\} \quad (n = 1, 2, 3, \dots)\,, \tag{7}$$

which can be rewritten at $\sigma = 0$ in the compact σ-dependent form

$$\Gamma_\mu \equiv \sum_{n=0}^{\infty} \frac{\sigma^n}{n!} \gamma_\mu^{(n)}|_0 = \Pi_{+\mu}{}^\nu I_{-\nu}(\sigma) + \Pi_{-\mu}{}^\nu I_{+\nu}(-\sigma) + \pi_\alpha(-\sigma)\Psi_\mu^\alpha + \bar\Psi_\mu^\alpha \bar\pi_\alpha(\sigma). \tag{8}$$

Similarly, from conditions $(\theta^\alpha - \bar\theta^\alpha)\big|_0 = 0$ and $(\pi_\alpha - \bar\pi_\alpha)\big|_0 = 0$, we get

$$\Gamma^\alpha(\sigma) = \Theta^\alpha(\sigma) - \bar\Theta^\alpha(\sigma)\,, \quad \Gamma_\alpha^\pi(\sigma) \equiv \pi_\alpha(-\sigma) - \bar\pi_\alpha(\sigma)\,, \tag{9}$$

where

$$\begin{aligned}\Theta^\alpha(\sigma) = \theta^\alpha(-\sigma) - \Psi_\mu^\alpha \tilde q^\mu(\sigma) - \frac{1}{2\kappa} F^{\alpha\beta} \int_0^\sigma d\sigma_1 P_s \bar\pi_\beta \\ + \frac{1}{2\kappa} G^{\mu\nu} \Psi_\mu^\alpha \int_0^\sigma d\sigma_1 P_s (I_{+\nu} + I_{-\nu}),\end{aligned} \tag{10}$$

$$\begin{aligned}\bar\Theta^\alpha(\sigma) = \bar\theta^\alpha(\sigma) + \bar\Psi_\mu^\alpha \tilde q^\mu(\sigma) + \frac{1}{2\kappa} F^{\beta\alpha} \int_0^\sigma d\sigma_1 P_s \pi_\beta \\ + \frac{1}{2\kappa} G^{\mu\nu} \bar\Psi_\mu^\alpha \int_0^\sigma d\sigma_1 P_s (I_{+\nu} + I_{-\nu})\,.\end{aligned} \tag{11}$$

We introduced new variables, symmetric and antisymmetric under world-sheet parity transformation $\Omega : \sigma \to -\sigma$. For bosonic variables we use the standard notation

$$q^\mu(\sigma) = P_s x^\mu(\sigma)\,, \tilde q^\mu(\sigma) = P_a x^\mu(\sigma)\,, p_\mu(\sigma) = P_s \pi_\mu(\sigma)\,, \tilde p_\mu(\sigma) = P_a \pi_\mu(\sigma)\,,$$

while for fermionic ones we use the projectors on σ symmetric and antisymmetric parts

$$P_s = \frac{1}{2}(1 + \Omega)\,, \quad P_a = \frac{1}{2}(1 - \Omega)\,. \tag{12}$$

From $\{H_c\,, \Gamma_A\} = \Gamma'_A \approx 0\,, \Gamma_A = (\Gamma_\mu\,, \Gamma^\alpha\,, \Gamma_\alpha^\pi)$, it follows that there are no more constraints in the theory. For practical reasons we will find the algebra of the constraints ${}^\star\Gamma_A = (\Gamma_\mu\,,\Gamma'^\alpha\,,\Gamma^\pi_\alpha)$ instead of Γ_A. It has the form

$$\{{}^\star\Gamma_A\,,{}^\star\Gamma_B\} = M_{AB}\delta'\,, \tag{13}$$

where the supermatrix M_{AB} is given by the expression

$$M_{AB} = \left(\begin{array}{c|cc} -\kappa G^{eff}_{\mu\nu} & -2(\Psi_{eff})^\gamma_\mu & 0 \\ \hline -2(\Psi_{eff})^\alpha_\nu & \frac{1}{\kappa}F^{\alpha\gamma}_{eff} & -2\delta^\alpha{}_\delta \\ 0 & -2\delta_\beta{}^\gamma & 0 \end{array}\right)\,. \tag{14}$$

Here we obtain effective background fields

$$G^{eff}_{\mu\nu} = G_{\mu\nu} - 4B_{\mu\rho}G^{\rho\lambda}B_{\lambda\nu}\,, \quad (\Psi_{eff})^\alpha_\mu = \frac{1}{2}\Psi^\alpha_{+\mu} + B_{\mu\rho}G^{\rho\nu}\Psi^\alpha_{-\nu}\,,$$
$$F^{\alpha\beta}_{eff} = F^{\alpha\beta}_a - \Psi^\alpha_{-\mu}G^{\mu\nu}\Psi^\beta_{-\nu}\,, \tag{15}$$

where we introduced useful notation

$$\Psi^\alpha_{\pm\mu} = \Psi^\alpha_\mu \pm \bar\Psi^\alpha_\mu\,, \quad F^{\alpha\beta}_s = \frac{1}{2}(F^{\alpha\beta} + F^{\beta\alpha})\,, \quad F^{\alpha\beta}_a = \frac{1}{2}(F^{\alpha\beta} - F^{\beta\alpha})\,. \tag{16}$$

This is supersymmetric generalization of Seiberg and Witten open string metric, $G^{eff}_{\mu\nu}$, because all effective fields contain bilinear combinations of Ω odd fields.

The superdeterminant $s\det M_{AB}$ is proportional to $\det G^{eff}$. Because we assume that effective metric G^{eff} is nonsingular, we conclude that all constraints ${}^\star\Gamma_A$ are of the second class.

3. Solution of the constraints and noncommutativity

From $\Gamma_\mu = 0$, $\Gamma^\alpha = 0$ and $\Gamma^\pi_\alpha = 0$, we obtain

$$x^\mu(\sigma) = q^\mu - 2\Theta^{\mu\nu}\int_0^\sigma d\sigma_1 p_\nu + 2\Theta^{\mu\alpha}\int_0^\sigma d\sigma_1 p_\alpha\,, \quad \pi_\mu = p_\mu\,, \tag{17}$$

$$\theta^\alpha(\sigma) = \Phi^\alpha(\sigma) + \frac{1}{2}\tilde\xi^\alpha\,, \quad \pi_\alpha = p_\alpha + \tilde p_\alpha\,, \tag{18}$$

$$\bar\theta^\alpha(\sigma) = \Phi^\alpha(\sigma) - \frac{1}{2}\tilde\xi^\alpha\,, \quad \bar\pi_\alpha = p_\alpha - \tilde p_\alpha\,, \tag{19}$$

where

$$\Phi^\alpha(\sigma) = \frac{1}{2}\,\xi^\alpha - \Theta^{\mu\alpha}\int_0^\sigma d\sigma_1 p_\mu - \Theta^{\alpha\beta}\int_0^\sigma d\sigma_1 p_\beta\,, \quad \tilde\xi^\alpha \equiv P_a(\theta^\alpha - \bar\theta^\alpha)\,,$$

$$\frac{1}{2}\,\xi^\alpha \equiv P_s\theta^\alpha = P_s\bar\theta^\alpha\,, \quad p_\alpha \equiv P_s\pi_\alpha = P_s\bar\pi_\alpha\,, \quad \tilde p_\alpha \equiv P_a\pi_\alpha = -P_a\bar\pi_\alpha\,,$$

and

$$\Theta^{\mu\nu} = -\frac{1}{\kappa}\,(G^{-1}_{eff}BG^{-1})^{\mu\nu}\,, \quad \Theta^{\mu\alpha} = 2\Theta^{\mu\nu}(\Psi_{eff})^\alpha_\nu - \frac{1}{2\kappa}\,G^{\mu\nu}\psi^\alpha_{-\nu}\,, \tag{20}$$

$$\Theta^{\alpha\beta} = \frac{1}{2\kappa}\,F_s^{\alpha\beta} + 4(\Psi_{eff})^\alpha_\mu\Theta^{\mu\nu}(\Psi_{eff})^\beta_\nu - \frac{1}{\kappa}\,\Psi^\alpha_{-\mu}(G^{-1}BG^{-1})^{\mu\nu}\Psi^\beta_{-\nu} + \frac{G^{\mu\nu}}{\kappa}\left[\Psi^\alpha_{-\mu}(\Psi_{eff})^\beta_\nu + \Psi^\beta_{-\mu}(\Psi_{eff})^\alpha_\nu\right]. \tag{21}$$

Using the solutions of the constraints (17)-(19) and the basic Poisson algebra, after removing the center of mass variables, we get the noncommutativity relations

$$\{x^\mu(\sigma)\,,x^\nu(\bar\sigma)\} = \Theta^{\mu\nu}\Delta(\sigma+\bar\sigma)\,, \tag{22}$$

$$\{x^\mu(\sigma)\,,\theta^\alpha(\bar\sigma)\} = -\frac{1}{2}\Theta^{\mu\alpha}\Delta(\sigma+\bar\sigma)\,, \quad \{\theta^\alpha(\sigma)\,,\bar\theta^\beta(\bar\sigma)\} = \frac{1}{4}\Theta^{\alpha\beta}\Delta(\sigma+\bar\sigma)\,. \tag{23}$$

The function $\Delta(\sigma+\bar\sigma)$ is nonzero only at string endpoints

$$\Delta(x) = \begin{cases} -1 & \text{if } x=0 \\ 0 & \text{if } 0<x<2\pi\,, \\ 1 & \text{if } x=2\pi \end{cases} \tag{24}$$

and we conclude that interior of the string is commutative, while string endpoints are noncommutative.

4. Supersymmetry of noncommutativity relations

The action of initial theory (1) is invariant under global $N=2$ supersymmetry with transformations of supercoordinates

$$\delta x^\mu = \bar\epsilon^\alpha\Gamma^\mu_{\alpha\beta}\theta^\beta + \epsilon^\alpha\Gamma^\mu_{\alpha\beta}\bar\theta^\beta\,, \quad \delta\theta^\alpha = \epsilon^\alpha\,, \quad \delta\bar\theta^\alpha = \bar\epsilon^\alpha\,, \tag{25}$$

and background fields

$$\delta G_{\mu\nu} = \epsilon^\alpha_+\Gamma_{[\mu\,\alpha\beta}\Psi^\beta_{+\nu]} - \epsilon^\alpha_-\Gamma_{[\mu\,\alpha\beta}\Psi^\beta_{-\nu]}\,, \delta B_{\mu\nu} = \epsilon^\alpha_+\Gamma_{[\mu\,\alpha\beta}\Psi^\beta_{-\nu]} - \epsilon^\alpha_-\Gamma_{[\mu\,\alpha\beta}\Psi^\beta_{+\nu]}\,,$$

$$\delta\Psi^\alpha_{+\mu} = -\frac{1}{16}\,\epsilon^\beta_-\Gamma_{\mu\,\beta\gamma}F_s^{\gamma\alpha} - \frac{1}{16}\,\epsilon^\beta_+\Gamma_{\mu\,\beta\gamma}F_a^{\gamma\alpha}\,,$$

$$\delta\Psi^\alpha_{-\mu} = \frac{1}{16}\,\epsilon^\beta_+\Gamma_{\mu\,\beta\gamma}F_s^{\gamma\alpha} + \frac{1}{16}\,\epsilon^\beta_-\Gamma_{\mu\,\beta\gamma}F_a^{\gamma\alpha}\,, \tag{26}$$

$$\delta A^{(0)}=0\,,\quad \delta A^{(2)}_{\mu\nu}=-\epsilon^\alpha_+\Gamma_{[\mu\,\alpha\beta}\Psi^\beta_{+\nu]}-\epsilon^\alpha_-\Gamma_{[\mu\,\alpha\beta}\Psi^\beta_{-\nu]}+A^{(0)}\delta B_{\mu\nu}\,,$$
$$\delta A^{(4)}_{\mu\nu\rho\sigma}=2\epsilon^\alpha_+\Gamma_{[\mu\nu\rho\,\alpha\beta}\Psi^\beta_{-\sigma]}+2\epsilon^\alpha_-\Gamma_{[\mu\nu\rho\,\alpha\beta}\Psi^\beta_{+\sigma]}+6A^{(2)}_{[\mu\nu}\delta B_{\rho\sigma]}\,.$$

Here we used notation

$$\epsilon^\alpha_\pm=\epsilon^\alpha\pm\bar\epsilon^\alpha=const.\,,\qquad \Gamma_{\mu_1\mu_2\ldots\mu_k}\equiv\Gamma_{[\mu_1}\Gamma_{\mu_2}\ldots\Gamma_{\mu_k]}\,,\tag{27}$$

and [ ] in the subscripts of the fields mean antisymmetrization of space-time indices between brackets. The connection between potentials $A^{(0)}$, $A^{(2)}_{\mu\nu}$ and $A^{(4)}_{\mu\nu\rho\sigma}$ and RR field strength $F^{\alpha\beta}$ is given in [7].

The truncation from $N=2$ to $N=1$ supersymmetry we can realize omitting transformations for $G_{\mu\nu}$, $\Psi_{+\mu}$ and F_a [10]. The rest fields make $N=1$ supermultiplet with transformation rules

$$\delta B_{\mu\nu}=\epsilon^\alpha_+\Gamma_{[\mu\,\alpha\beta}\Psi^\beta_{-\nu]}\,,\quad \delta\Psi^\alpha_{-\mu}=\frac{1}{16}\epsilon^\beta_+\Gamma_{\mu\,\beta\gamma}F^{\gamma\alpha}_s\,,\quad \delta F^{\alpha\beta}_s=0\,.\tag{28}$$

Now we can find the supersymmetric transformations of the coefficients, $\Theta^{\mu\nu}$, $\Theta^{\mu\alpha}$ and $\Theta^{\alpha\beta}$, multiplying the momenta in the solutions of boundary conditions. From

$$\delta x^\mu(\sigma)=\frac12\epsilon^\alpha_+\Gamma^\mu_{\alpha\beta}\xi^\beta-2\delta\Theta^{\mu\nu}\int_0^\sigma d\sigma_1p_\nu-2\Theta^{\mu\nu}\int_0^\sigma d\sigma_1\delta p_\nu\,,\tag{29}$$
$$+2\delta\Theta^{\mu\alpha}\int_0^\sigma d\sigma_1p_\alpha+2\Theta^{\mu\alpha}\int_0^\sigma d\sigma_1\delta p_\alpha=\epsilon^\alpha_+\Gamma^\mu_{\alpha\beta}\theta(\sigma)^\beta-\frac12\epsilon^\alpha_+\Gamma^\mu_{\alpha\beta}\tilde\xi(\sigma)^\beta\,,$$

$$\delta\theta^\alpha(\sigma)=\frac12\epsilon^\alpha_+-\delta\Theta^{\mu\alpha}\int_0^\sigma d\sigma_1p_\mu$$
$$-\Theta^{\mu\alpha}\int_0^\sigma d\sigma_1\delta p_\mu-\delta\Theta^{\alpha\beta}\int_0^\sigma d\sigma_1p_\beta-\Theta^{\alpha\beta}\int_0^\sigma d\sigma_1\delta p_\beta=\frac12\epsilon^\alpha_+\,,\tag{30}$$

we obtain global $N=1$ SUSY transformations of the background fields

$$\delta\Theta^{\mu\nu}=\epsilon^\alpha_+\Gamma^{[\mu}_{\alpha\beta}\Theta^{\nu]\beta}\,,\quad \delta\Theta^{\mu\alpha}=-\frac12\epsilon^\beta_+\Gamma^\mu_{\beta\gamma}\Theta^{\gamma\alpha}\,,\quad \delta\Theta^{\alpha\beta}=0\,.\tag{31}$$

Consequently, these fields are components of $N=1$ supermultiplet.

Using $N=1$ supersymmetry transformations of SUSY coordinates (25) and background fields (31), we can easily prove that noncommutativity relations are connected by supersymmetry transformations. The SUSY transformation of (22)

$$\epsilon^\alpha_+\Gamma^{[\mu}_{\alpha\beta}\left\{x^{\nu]},\theta^\beta\right\}=-\frac12\epsilon^\alpha_+\Gamma^{[\mu}_{\alpha\beta}\Theta^{\nu]\beta}\Delta(\sigma+\bar\sigma)\,,\tag{32}$$

produces the first relation in (23). Similarly, SUSY transformation of the first relation in (23)

$$\epsilon_+^\beta \Gamma^\mu_{\beta\gamma} \{\theta^\gamma, \theta^\alpha\} = \frac{1}{4} \epsilon_+^\beta \Gamma^\mu_{\beta\gamma} \Theta^{\gamma\alpha} \Delta(\sigma + \bar{\sigma}) \,, \tag{33}$$

produces the second relation in (23).

5. Type IIB theory and T-duality

We suppose that action (1) has a global shift symmetry in all x^μ directions. So, we have to introduce gauge fields $v^\mu_\pm$ and make a change

$$\partial_\pm x^\mu \to D_\pm x^\mu \equiv \partial_\pm x^\mu + v^\mu_\pm \,. \tag{34}$$

For the fields $v^\mu_\pm$ we introduce additional term in the action

$$S_{gauge}(y, v_\pm) = \frac{1}{2}\kappa \int_\Sigma d^2\xi y_\mu (\partial_+ v^\mu_- - \partial_- v^\mu_+) \,, \tag{35}$$

which produces vanishing of the field strength $\partial_+ v^\mu_- - \partial_- v^\mu_+$ if we vary with respect to the Lagrange multipliers y_μ. The full action has the form

$$S^\star(x, y, v_\pm) = S(D_\pm x) + S_{gauge}(y, v_\pm) \,. \tag{36}$$

Let us note that on the equations of motion for y_μ we have $v^\mu_\pm = \partial_\pm x^\mu$ and the original dynamics survives unchanged.

Now we can fix x^μ to zero and obtain the action quadratic in the fields $v^\mu_\pm$ which can be integrated out classically. On the equations of motion for $v^\mu_\pm$ we obtain expressions for these gauge fields in terms of y_μ and momenta

$$v^\mu_+ = -2 \left(\frac{2}{\kappa} \pi_\alpha \Psi^\alpha_\nu + \partial_+ y_\nu \right) (G^{-1}_{eff} \Pi_- G^{-1})^{\nu\mu} \,, \tag{37}$$

$$v^\mu_- = 2 (G^{-1}_{eff} \Pi_- G^{-1})^{\mu\nu} \left(\frac{2}{\kappa} \bar{\Psi}^\alpha_\nu \bar{\pi}_\alpha + \partial_- y_\nu \right) \,. \tag{38}$$

Substituting them in the action $S^\star$ we obtain the dual action from which we read the dual background fields

$$^\star G^{\mu\nu} = (G^{-1}_{eff})^{\mu\nu} \,, \quad ^\star B^{\mu\nu} = \kappa \Theta^{\mu\nu} = -(G^{-1}_{eff} B G^{-1})^{\mu\nu} \,, \tag{39}$$

$$^\star \Psi^{\mu\alpha} = \left[2\kappa \Theta^{\mu\nu} - (G^{-1}_{eff})^{\mu\nu} \right] \Psi^\alpha_\nu \,, \quad ^\star \bar{\Psi}^{\mu\alpha} = \left[2\kappa \Theta^{\mu\nu} + (G^{-1}_{eff})^{\mu\nu} \right] \bar{\Psi}^\alpha_\nu \,, \tag{40}$$

$$^\star F^{\alpha\beta}_s = 2\kappa \Theta^{\alpha\beta} \,, \quad ^\star F^{\alpha\beta}_a = F^{\alpha\beta}_{eff} + 4 (\Psi_{eff})^\alpha_\mu (G^{-1}_{eff})^{\mu\nu} (\Psi_{eff})^\alpha_\nu \,. \tag{41}$$

It is easy to see that

$$^\star \Psi^{\mu\alpha}_+ = 2\kappa \Theta^{\mu\alpha} \,, \, ^\star \Psi^{\mu\alpha}_- = -2 (G^{-1}_{eff})^{\mu\nu} (\Psi_{eff})^\alpha_\mu \,. \tag{42}$$

6. Concluding remarks

In this paper we considered noncommutativity properties and related supersymmetry transformations of $D9$ brane in type IIB superstring theory. We used the pure spinor formulation of the theory introduced in Refs.[2, 9].

We treated all boundary conditions at string endpoints as canonical constraints. Solving the second class constraints we obtain the initial coordinates x^μ, θ^α and $\bar\theta^\alpha$ in terms of effective ones, q^μ, ξ^α and $\tilde\xi^\alpha$ and momenta p_μ and p_α.

The presence of momenta is a source of noncommutativity (22)-(23). The result of the present paper can be considered as a supersymmetric generalization of the result obtained for bosonic string [8]. Beside $B_{\mu\nu}$, its superpartners $\Psi^\alpha_{-\mu}$ and $F_s^{\alpha\beta}$ are also a source of noncommutativity. In special case, when $\Psi^\alpha = \bar\Psi^\alpha_\mu$, the noncommutativity relations (22)-(23) correspond to the relations of Ref.[9].

The $N = 2$ supermultiplet of T-dual theory is split in two $N = 1$ supermultiplets: the first, Ω even one, $(G^{eff}_{\mu\nu}, (\Psi_{eff})^\alpha_\mu, F^{\alpha\beta}_{eff})$, represents the background fields of the effective theory (initial theory on the solution of the boundary conditions) [7], while the second, Ω odd one, $(\Theta^{\mu\nu}, \Theta^{\mu\alpha}, \Theta^{\alpha\beta})$, contains the noncommutativity parameters.

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