Kerr-Newman Solution as Gravitating Soliton: Electromagnetic Excitations

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Abstract

Electromagnetic (EM) excitations of the Kerr- Newman (KN) soliton model are considered. The model of regular source of the KN solution was suggested recently in [3] in the form of a rotating domain wall bubble, interior of which is filled by a superconducting matter modelled by the Higgs field. Regularization of the internal EM field is performed the Higgs effect, which expels the EM field to boundary of the bubble. The KN soliton takes the form of a disklike source which is compatible with the external EM KN field and metric. Inner structure of the source is similar to the models of Q-balls, oscillons and breathers. Using the Kerr-Schild formalism we obtain exact solutions for the EM excitations of the Kerr geometry, and show that excitations of the bubble source generate the beamlike pulses tending to singular pp-wave solutions in he far zone. We link regularization of the pp-wave beams with regularization of the Kerr singular ring, which allows us to estimate radius of the beam core.

1. Introduction: Regular Source of Kerr-Newman (KN) solution.

There are many evidences that black holes (BH) are akin to elementary particles [1]. In particular, the KN solution has $g = 2$ as that of the Dirac electron, [2]. The KN solution is the most natural generalization of the Coulomb one. It gives us a consistent with gravity classical model of spinning electron. Because of that, the consistent with gravity approach to regularization of the KN solution may shed a new light on the problems of divergencies in QED and open a new way to quantum gravity.

The problem of regularization of the black holes (BH) and in particular of the KN solution has a long story. An outcome was given recently in [3] (it will be referred henceforth as I), where a soliton model of the electron was given as a regular KN solution, completed by a system of the chiral fields

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which produce a regular internal pseudovacuum state. Since spin of the electron is very high, the black-hole horizons of the KN solution disappear, opening a naked Kerr singular ring which is a branch line of the spacetime. There appears a holographic *two-sheeted structure* of the Kerr geometry [4, 5, 6]. To avoid this two-sheetedness, the Kerr ring should be covered by a regular matter source.

A very long treatment of the problem of KN source (Newman, Israel, Hamity, A.B.,Lopez, Krasinski and others...) has been evolved into the model of a rotating domain wall bubble, interior of which is filled by a superconducting matter which expels the electromagnetic(EM) KN field and currents to the boundary of the bubble.

To achieve a compatibility with the external KN geometry, the oblate disk-like form of the bubble has to be used. So, boundary of the disk corresponds to the surface $r = r_0 = const.$ of the oblate ellipsoidal coordinate system r, θ of the KN geometry. Consistency of the model dictates that interior of

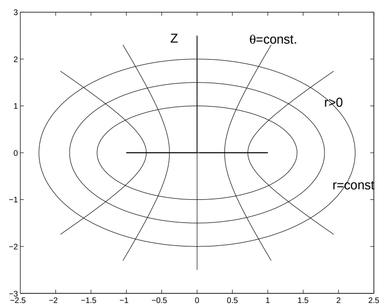

Figure 1: The oblate coordinate system.

the model has to be flat and filled by a system of the chiral fields forming a false vacuum inside the bubble and a (domain wall) phase transfer to the external vacuum state consistent with the external KN solution. One of the chiral fields has to be the Higgs field. It takes inside the bubble the non-zero value $\Phi = \Phi_0 \exp\{i\chi\}$ and interacts with the KN EM field $F_{\mu\nu} = A_{\mu,\nu} - A_{\nu,\mu}$, which is described by the Higgs Lagrangian,

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} \mathcal{D}_\mu \Phi \bar{\mathcal{D}}^\mu \bar{\Phi} + V, \tag{1}$$

where $\mathcal{D}_\mu = \nabla_\mu + ieA_\mu$. The Higgs mechanism (LG model) models superconductivity similar to the Nielsen-Olesen model of superconducting string [7]. The resulting equations are

$$\Box A_\mu = I_\mu = e|\Phi|^2 (\chi,_\mu + eA_\mu). \tag{2}$$

Inside the bubble, $|\Phi| > 0, \quad I_\mu = 0$, we have

$$\Box A_\mu = 0, \quad \chi,_\mu + eA_\mu = 0, \tag{3}$$

which shows that gradient of the phase of the Higgs field $\chi_{,\mu}$ "eats up" the KN electromagnetic(EM) field A_μ, (22), expelling the field strength and currents to the boundary of the bubble. Therefore, the Higgs effect forms a superconducting bubble core of the KN solution, which regularizes the KN EM field. The KN gravitational field is regularized by the system of chiral fields, which provide the phase transition from the external KN field to a flat interior of the bubble. For parameters of an electron, the bubble core has the Compton radius $a = \frac{\hbar}{2m}$ forming an extremely oblated rotating disk. Thickness of the disk is equal to the "classical electron size" $r_0 = e^2/2m$. So, the degree of oblateness is $r_0/a \sim \frac{e^2}{\hbar} \sim 137^{-1}$. The KN vector potential A_μ (22) takes at the boundary of the border of the bubble a finite maximal value

$$\max |A^\mu| = \frac{2m}{e} k^\mu \quad (4)$$

which forms a closed circular string. In accordance with (3), the time component of the EM field $|A_0| = \frac{2m}{e}$ should be matched with $\chi_{,0}$ which determines frequency of the Higgs field $\omega = 2m$. Since k^m is the null field,$k^\mu = (1, \vec{k})$ and $|\vec{k}| = 1$, the spacelike and timelike components are equal, $|\vec{A}| = |A_0| = \frac{2}{m} e$. In the same time, due to twisting form of the Kerr principal null directions, $\vec{A}$ turns out to be tangent to the border of the bubble and forms a closed circular component $A_\phi = \frac{2}{m} e$ along the border. Its amplitude also has to be matched with a periodicity of the Higgs field. As a result, the loop integral $S^{(loop)} = \oint e A_\phi^{(loop)} d\phi$ turns out to be quantized, leading to the quantum condition

$$S^{(loop)} = -4\pi ma = 2\pi n, \; n = 1, 2, 3, ... \quad (5)$$

which, due to the Kerr relation $J = ma$, implies quantization of the angular momentum J of the regularized KN solution, $|J| = ma = n/2, \; n = 1, 2, 3, ...$ We arrive at the following conclusions:

(i) the KN EM field of the model is to be regularized and its vector potential is finite, $|A_\mu| \le \frac{2}{m} e$,

(ii) angular momentum of the bubble source is to be quantized, $J = ma = n/2, \; n = 1, 2, 3, ...$

(iii) the Higgs field of the bubble forms a coherent pseudo-vacuum state oscillating with the frequency $\omega = \dot{\chi} = 2m$, like the spinning versions of the spinning Q-balls, oscillons [8, 9] and the bosonic star models [10].

For consistency of the KN electron model, [11, 12, 13, 14], the bubble-disk should have the Compton radius. In principle, it agrees with the predicted by QED radius of the region of virtual photons.[1] However, the KN solitonic

[1]The Compton size of the electron was also suggested by Compton [15] from the analysis of experimental data on a soft scattering of the electron on the metallic surfaces.

model of electron has a new principal feature, which is not predicted by QED: the Higgs field in the bubble core is to be coherently oscillating. It means that the disklike bubble is not a simple cloud of virtual photons, but it should be considered as an integral element of the electron structure.

The electromagnetic field and currents in a superconducting matter have a 'penetration depth' $\delta \sim \frac{1}{e|\Phi|} = 1/m_v$, [7], which forms a thin surface layer where (3) is broken, and the deviations $A^{(\delta)}_\mu = A_\mu - A^{(in)}_\mu$ obey the equation $\Box A^{(\delta)}_\mu = m_v^2 A^{(\delta)}_\mu$. It shows the appearance of circular currents at the boundary of the disklike core. These currents strongly increase in the equatorial plane forming a circular string, near the former Kerr singular ring. The EM excitations of the Kerr geometry is related with traveling waves along the border of the disk, which was described in the very old Kerr's microgeon model [11], and is similar to excitations of the Sen heterotic string model, [16].[2]

The considered in I case of the regularized KN solution corresponds to a stationary solitonic background, without excitations. Meanwhile, the wave excitations of the Kerr electron (the stringy traveling waves) [11, 13, 17] represent important part of its structure, since they are to be connected with generation of de Brougle waves [13, 14]. It was shown in [5, 4], that the EM excitations of the KN geometry generate an infinite class of the outgoing singular beam pulses tending asymptotically to singular pp-waves [18, 19] propagating in different angular directions along twistor null rays

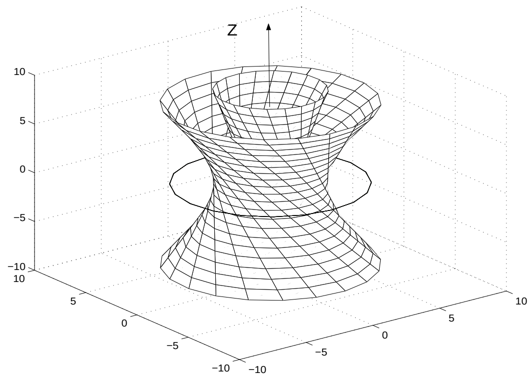

Figure 2: The Kerr singular ring and the twistor null lines of the Kerr congruence.

of the Kerr congruence. There appears a question, how interplay these solutions with the Higgs field of the regular KN source. We obtain that amplitude of the beamlike solutions is strongly controlled by the Higgs core of the regular KN source.

Next, we derive the exact expression for the nonstationary EM excita-

[2]It has also been supported by many other treatments, in particular, in I and in [13, 17].

tions of the Kerr geometry in terms of the complex vector potential. This derivation is represented in App.D, however, it is rather cumbersome and demands recalculation of many intermediate relations of Kerr-Schild formalism, which are given in the App.A,B,C. For the treatment of *the gravitational sector* of the KN soliton and the the phase transition between the inner and external vacua controlled by the *chiral structure* of the model, we refer the readers to the paper I . We use the technics and notations of the fundamental work by Debney, Kerr and Schild, [2], which will be referred henceforth as DKS.

2. Basics of the Kerr-Schild formalism

The Kerr metric has the Kerr-Schild form

$$g_{\mu\nu} = \eta_{\mu\nu} + 2he^3_\mu e^3_\nu, \tag{6}$$

where $\eta_{\mu\nu} = diag\{-1, 1, 1, 1\}$ is the metric of auxiliary Minkowski background $x^\mu = (t, x, y, z) \in M^4$.

The vector field e^3_μ is described in terms of the null Cartesian coordinates $(u,\ v,\ ,\zeta,\ \bar\zeta)$ and has the form

$$e^3 = du + \bar Y d\zeta + Y d\bar\zeta - Y\bar Y dv. \tag{7}$$

The vector field e^3 is tangent to Kerr's principal null congruence (PNC) $\mathcal{K}$ which is determined by the complex function $Y \equiv Y(x^\mu),\ x^\mu \in \mathbb{M}^4$. In fact, the function Y is asymptotically a projective angular coordinate on an infinite celestial sphere,

$$Y = e^{i\phi_K} \tan(\theta/2), \tag{8}$$

however, in the near zone the angular coordinates turn in the oblate spheroidal ones with a very specific Kerr angular coordinate ϕ_K. The relation between the oblate coordinates and the Cartesian ones is

$$x + iy = (r + ia) \exp\{i\phi_K\} \sin\theta, \quad z = r\cos\theta, \quad \rho = r - t. \tag{9}$$

The oblate coordinate r is adapted to a two-sheeted structure of the Kerr spacetime and covers $\mathbb{M}^4$ twice, taking the positive, $r > 0$, and negative, $r < 0$, values in agreement with a two-sheeted structure of the Kerr spacetime. The e^3 direction is completed to the aligned with PNC null tetrad e^a,

$$e^1 = d\zeta - Y dv,\ e^2 = d\bar\zeta - \bar Y dv,\ e^4 = dv + he^3. \tag{10}$$

The dual tetrad is obtained by permutations $e_1 = e^2,\ e_2 = e^1,\ e_3 = e^4,\ e_4 = e^3$. The corresponding directional derivatives $\partial_a \equiv\ (\ )_{,a} \equiv e^\mu_a \partial_\mu(\ )$ are following

$$\begin{aligned} &\partial_1 = \partial_\zeta - \bar Y \partial_u, \quad \partial_2 = \partial_{\bar\zeta} - Y\partial_u, \\ &\partial_3 = \partial_u - h\partial_4, \quad \partial_4 = \partial_v + Y\partial_\zeta + \bar Y \partial_{\bar\zeta} - Y\bar Y \partial_u. \end{aligned} \tag{11}$$

It is known [20] that metric of the Kerr BH solution may be described either in the terms of the outgoing PNC or in the terms of the ingoing one, but not at the use of the both of them simultaneously. If a time-dependent EM field is present, the freedom of choice is lost. The wave EM field is associated with a retarded field, which selects the use of outgoing congruence. This point demands some extra attention, since the choice of PNC influences on the signs of many expressions. The used in DKS null Cartesian coordinates $2^{\frac{1}{2}}\zeta = x + iy$, $2^{\frac{1}{2}}\bar{\zeta} = x - iy$, $2^{\frac{1}{2}}u = z + t$, $2^{\frac{1}{2}}v = z - t$ are adapted to an ingoing PNC. The simplest case to transfer to an outgoing congruence is to permute the time-dependence of the functions u and v, and we use the following notations

$$2^{\frac{1}{2}}\zeta = x + iy, \qquad 2^{\frac{1}{2}}\bar{\zeta} = x - iy, \qquad 2^{\frac{1}{2}}u = z - t, \qquad 2^{\frac{1}{2}}v = z + t. \quad (12)$$

corresponding to a future-oriented congruence.

The classical solutions should have the future-oriented Killing vector $K^\mu, K^0 > 0$. In DKS the Killing operator $\hat{K} = K^\mu \partial_\mu$ is considered in null coordinates

$$\hat{K} = c\partial_u + \bar{q}\partial_\zeta + q\partial_{\bar{\zeta}} - p\partial_v. \quad (13)$$

The stationary Kerr-Schild (KS) metrics should be invariant with respect to the real Killing directions,

$$\hat{K} g^{\mu\nu} = 0. \quad (14)$$

In particular, $g^{\mu\nu}$ depends on e^3, (7), which is determined by function $Y(x^\mu)$. While, the function $Y(x^\mu)$ is determined by the *Kerr theorem* (see App.B.) for the stationary, geodesic and shear free congruences. A generalized condition of stationarity takes the form

$$\hat{K} Y = \hat{K} \bar{Y} = 0. \quad (15)$$

From (11) one sees that $\partial_1 \hat{K}\bar{Y} = \hat{K}\bar{Y},_1$, but $Y,_1 = Z$ which yields $\hat{K}Z = 0$. Performed in DKS integration showed that general form of the function h in terms of Y and Z is $h = M(Y,\bar{Y})(Z + \bar{Z})/2 + B(Y,\bar{Y})(Z\bar{Z})$. It shows that $\hat{K}h = 0$, and consequently (15) implies (14).

Function Z is inversely proportional to the complex radial distance $\tilde{r}$,

$$\tilde{r} = r + i\cos\theta = -PZ^{-1}, \quad (16)$$

where the function

$$P = c + \bar{q}\bar{Y} + qY + pY\bar{Y} \quad (17)$$

is to be related with the Killing vector (13). In particular, for the future-oriented Kerr solution at rest, $K^\mu = (1,0,0,0)$, $\hat{K} = \partial_t$ and we have $c = p = -1/\sqrt{2}$, $q = \bar{q} = 0$, which yields

$$P = -(1 + Y\bar{Y})/\sqrt{2}\ . \quad (18)$$

The sign of P differs from the given in DKS.[3] In this case, it is convenient to replace e^3_μ by the normalized vector field

$$k_\mu dx^\mu = P^{-1} e^3 \tag{19}$$

having $k_0 = 1$. Then, the KN metric may be represented in the form

$$g_{\mu\nu} = \eta_{\mu\nu} + 2H k_\mu k_\nu, \tag{20}$$

where $H = hP^3$ takes the simple form

$$H = (mr - e^2)/(r^2 + a^2 \cos^2\theta). \tag{21}$$

Vector potential of the KN solution is given by

$$A^\mu_{KN} = Re \frac{e}{r + ia\cos\theta} k^\mu, \tag{22}$$

where k^μ may also be represented in the form (see App.B,(58))

$$k_\mu dx^\mu = dr - dt - a \sin^2\theta d\phi_K. \tag{23}$$

3. Electromagnetic excitations of the Kerr-Schild background

The obtained in DKS general solutions for electromagnetic field on the Kerr-Schild background are described by the auto-dual components $\mathcal{F}_{ab}$ with respect to the tetrad e^a, given by Eqs. (10). The field has only two nonzero complex components,

$$\mathcal{F}_{12} = \mathcal{F}_{34} = AZ^2; \quad \mathcal{F}_{31} = \gamma Z - (AZ),_1 \ . \tag{24}$$

The real electromagnetic field F_{ab} is determined by the relations

$$F_{41} = F_{42} = 0; \quad F_{12} + F_{34} = \mathcal{F}_{12}; \quad 2F_{31} = \mathcal{F}_{31} \ . \tag{25}$$

For the case $\gamma = 0$, integration was completed in DKS. The solutions are given by $A = \frac{\psi}{P^2}$, where ψ may be an arbitrary analytic function of Y, $\psi = \psi(Y)$. The case $\psi = e = const.$ corresponds to KN solution. Other solutions have paid attention only recently. It was shown in [4], that the function $\psi(Y) = \sum_i q_i/(Y - Y_i)$ corresponds to a *linear superposition of singular beams*, which are supported by the straight twistor lines of the Kerr congruence, Fig.2, and the beams break the horizon in angular directions $Y_i = \exp\{i\phi\} \tan\frac{\theta}{2}$.

[3] To specify the signs we recalculate many expressions of the KS formalism in Appendixes.

The solutions with $\gamma \neq 0$ were obtained in [5]. They are nonstationary and depend on an extra complex retarded-time coordinate τ, see App.B,C.,

$$A = \sum_i \psi_i(Y,\tau)/P^2, \quad \psi_i(Y,\tau) = q_i(\tau)/(Y - Y_i). \tag{26}$$

Any EM excitation of the Kerr geometry generates the beam pulses. In App.D we show that (24) and (25) may be expressed via complex vector potential

$$\mathcal{A} = AZe^3 + \bar{\chi} dY, \tag{27}$$

where $\bar{\chi} = \int Ad\bar{Y}$, and Y and τ being kept constant in this integration. Together with (61) and (73), this eq. shows that the fields $\mathcal{A}$, dY and $d\tau$ are spanned by the null vectors e^1 and e^3, and therefore, they lie in the "left" complex null planes. The complex vector potential (27) and (26) describes infinite family of the pulsing EM beams propagating along the twistor null lines of the Kerr congruence. The i-th beam in the angular direction Y_i has $\psi_i(Y,\tau) = \frac{q_i(\tau)}{(Y-Y_i)}$, and the function

$$\bar{\chi}_i = \int_{Y,\tau=const.} d\bar{Y} \frac{q_i(\tau)}{P^2(Y - Y_i)} = \frac{-2q_i(\tau)}{Y_i(Y - Y_i)(1 + Y_i \bar{Y}_i)}. \tag{28}$$

Taking into account that $k_\mu = e^3_\mu/P$ and $\tilde{r} = -P/Z = r + ia\cos\theta$, we obtain for the complex potential caused by the i-th beam the expression

$$\mathcal{A}^{(i)} = \psi_i(Y,\tau)\{ \frac{1}{r + ia\cos\theta_i} k^{(i)}_\mu dx^\mu - \frac{\sqrt{2}dY}{P_i Y_i}\}, \tag{29}$$

where $P_i = (1 + Y_i \bar{Y}_i)/\sqrt{2}$.

4. Regularization of singularities

The expression (29) for complex potential is important, since it has the form which is close to the used in QED representation for quantum oscillators. The usual plane waves conflict with the general covariance of gravity, bringing also the problem of their infinite energy. Quantum theory uses widely the Fourier transform, which is also a source of the incompatibility with gravity. It seems reasonable that this conflict has to be overcome by a mutual convergence of these theories. From the gravity side, such a concession is related with a renunciation of the general covariance, which is brilliantly achieved in the KS form of metric which has an extremely fixed coordinate system adapted to the auxiliary Minkowski space-time. The concession from quantum side should apparently be related with the renunciation of the Fourier transform in its usual form. In this respect, resolution of the conflict arrives from twistors. On the one hand, the twistors are basic elements of the KS geometry, on the other hand is there is a twistor analog of the Fourier transform [28, 27, 6], in which the basic objects are the complex null planes. In the KS geometry they are the complex null planes spanned by the e^3 and e^1 null directions. They are retained to be flat planes in the curved KS space time, since the function h drops out of the metric (6) due to orthogonality of the directions, $e^3_\mu e^{3\mu} = 0, \quad e^3_\mu e^{1\mu} = 0$. The complex representation of the vector potential (29) is exactly spanned by the e^3 and e^1 null directions (see 61)) which shows that it lies in the twistor null plane, and this representation is the most close to description of quantum oscillators.

The expression (29) shows also that any elementary EM excitation of the KS geometry is related with excitations at least of two topologically coupled singular lines: one of them is the Kerr singular ring $\tilde{r} = r + ia\cos\theta = 0$, and the second line is the axial singularity caused by one of the poles of function ψ. Axial beams appear inavoidably in all the non-stationary solutions. For the stationary KN solution regularization of the Kerr singularity was considered in I, and briefly in the Introduction to this paper.

The non-stationary solutions, related with the EM excitations of the KS geometry, are accompanied by traveling waves for which these singularities play the role of waveguides. In the KS solutions these waveguides are formed by the twistor null lines of the Kerr congruence, and the associated traveling waves go in far zone to the well known in gravity pp-wave solutions, which turn out to be consistent with gravity analogs of the plane waves.

The Kerr singular ring plays also the role of a waveguide for the circular traveling waves generating the mass and spin of the electron, which was mentioned still in the old microgeon model [11]. In the regular KN soliton model considered in I, the waveguide is formed by the edge rim of the disklike KN source. As a consequence of the wave solutions (29) the circular and axial pp-waves are mutually connected and excitations of the KN geometry may be considered as a source of the exact pp-wave solutions.

The potential (29) does not fall off by $r \to \infty$. Near the i-th beam in the far zone, $|Y - Y_i| = \rho_i/2r$, where $\rho_i \approx r\delta\theta_i$, is the distance from the beam singularity. As a result, the r dependence cancels for the time and longitudinal components of the potential (proportional to $k^{(i)\infty} = (dt, dr)$), as well as for the transverse polarized components, proportional to $dY_i/Y_i = id\phi_i + \frac{1}{\sin\theta_i} d\theta_i \sim \frac{1}{r} dx^\mu_\perp$. As a back-reaction, the i-th singular EM pp-wave produces a singular action on the metric – a gravitational pp-wave, [18], of the KS form (6) with the asymptotically constant null direction $k_i = e^3_i/P_i$. One sees that the KS metric for a single beam in the far zone is determined by function $H = -\frac{4|q_i(\tau)|^2}{\rho_i^2}$ which also depends only on the distance from the beam ρ_i. Notice, that in the transverse to k_i plane, metric of the pp-wave solutions is flat, and the 'conical singularity' is absent. So, the pp-waves form also the tensionless (Schild's) strings reminiscent of the heterotic strings of the superstring theory [16]. The beams may be regularized by the Higgs field in the same manner as it was done in I for the KN bubble source. If the pp-wave beams were created by the KN source, the condition

(4) should be retained for the beams in far zone too, and the Higgs field will oscillate with the frequency $\omega = 2m$. The amplitude $|\mathcal{A}^{(i)}| = \frac{2m}{e}$, will also determine the periodicity of the Higgs field along the beam direction, $\chi,_r = max|A_\mu k^\mu| = \frac{2m}{e}$. The transversal component, forming circulation of the potential around the beam singularity, allows us to estimate radius of the beam core. Setting $|q| = e$, and using the relation $d\phi_i = Im \ \frac{d(Y-Y_i)}{Y-Y_i}$, we equate the loop integral

$$S^{(loop)} = e \oint Re \ \mathcal{A}_\phi d\phi \sim 2\pi e^2/\rho_{min} \tag{30}$$

with the incursion of the Higgs phase 2π, and obtain $\rho_{min} \sim e^2$, which is $\sim 137^{-1}$ in the Planck units $G = \hbar = c = 1$.

Regularization of the Kerr circular singularity was performed in I only for the stationary KN case. Appearance of the EM excitations impose essential constraints on this procedure. Origin of the obstacles is related with the leading term of the real vector potential (29) in the equatorial plane of the KN solution

$$Re \ \mathcal{A}|_{board} = \frac{1}{r_b} Re \ \psi(Y,\tau) \ k_\mu^{(i)} dx^\mu. \tag{31}$$

The KN solution has $\psi(Y,\tau) = const. = e$, which produce an uniform flow of the vector potential along the circular board of the KN bubble source, $\cos\theta = 0; \ r = r_b$, which allowed us to satisfy the condition (29) for the phase of the Higgs field,

$$\chi,_\mu + eRe \ \mathcal{A}_\mu|_{board} = 0. \tag{32}$$

For traveling waves we have a dependence ψ on Y, and τ. At the board od the disklike source, $Y|_{board} = \exp i\phi$, and $\tau|_{board} = t$, which leads to the flow with a variable amplitude, and moreover, to reverse direction of the flow. It seems that the matching $\chi,_\phi = -eRe \ \mathcal{A}_\phi|_{board}, \quad \chi,_0 = -eRe\mathcal{A}_0|_{board}$ is impossible. However, there is let-out – an extra parameter r_b, and the both these relations could be fulfilled if we set $A_0 = 2m/r_b$, or $r_b = 2m/A_0$. In this case, we can avoid reversing the sign by a special choice of Y-dependence. In particular, setting $Re\mathcal{A}_0|_{board} = \frac{2m}{e} Re \ (1+Y)|_{board} = \frac{m}{e}\cos^2(\theta/2)$, we obtain the EM excitation with a half-integer angular dependence. It is interesting this case may be considered as the simplest EM excitation of the charged KN solution. Effectively, it leads to a deformation of the bubble source, leading to breaking of its axial symmetry. However, investigation of these possibilities goes out of the frame of this paper, and it will be given elsewhere.

Appendix A: Complex representation of the DKS formalism

The Kerr and Kerr-Newman solutions may be considered as the fields generated by a complex source propagating in the complex spacetime $X_0^\mu \in \mathbb{CM}^4$

along a complex world-line (CWL) $X_0(\tau) = (\tau, 0, 0, -ia)$, parameterized by the complex time parameter τ, [21, 24, 13, 5]. It allows us to consider the KN solution as a real slice $x^\mu \in \mathbb{M}^4$ of some complex complex retarded-time construction with a complex radial distance

$$\tilde{r} = r + ia\cos\theta = \sqrt{(\vec{x} - \vec{x}_0)^2} = \sqrt{x^2 + y^2 + (z + ia)^2}. \qquad (33)$$

In the given by (12) null coordinates $x_0^n = (u_0, v_0, \zeta_0, \bar{\zeta}_0)$ we have

$$x_0^n = 2^{-1/2}(-ia - \tau,\ -ia + \tau,\ 0,\ 0). \qquad (34)$$

The unit timelike vector $\dot{x}_0^\mu = \partial_\tau x_0^\mu = (1, 0, 0, 0)$ will have the null Cartesian coordinates

$$\dot{u}_0 = -2^{-1/2}, \quad \dot{v}_0 = 2^{-1/2}, \quad \dot{\zeta}_0 = \dot{\bar{\zeta}} = 0, \qquad (35)$$

$$\partial_\tau x_0^n = 2^{-1/2}(-1, 1, 0, 0), \qquad (36)$$

and the tetrad direction e^3 will have the null components

$$e_n^3 = (1, -Y\bar{Y}, \bar{Y}, Y). \qquad (37)$$

Function

$$U = x^\mu e_\mu^3, \qquad (38)$$

(sometimes called the potential) plays important role of a real retarded time and determines a position of the front surfaces. Together with the complex angular coordinates $Y, \bar{Y}$ and the radial one r) it may be considered as the fourth spacetime coordinate, or one of the tetrad components (e^3) of the spacetime points x^μ. For the real x^μ and $Y = (\bar{Y})^*$ it is real. For the real points (12)

$$U = u + \bar{Y}\zeta + Y\bar{\zeta} - Y\bar{Y}v, \qquad (39)$$

and action of the operator (13) on U determines function

$$P = \hat{K}U = c + \bar{q}\bar{Y} + qY + pY\bar{Y}. \qquad (40)$$

On the CWL $x_0^n(\tau)$, the values $U_0 = x_0^n(\tau)e_n^3$ remain to be real for $Y = (\bar{Y})^*$, (see [24]). In the retarded-time construction $P = \hat{K}U = P_0 = \partial_\tau U_0$. The following from the Kerr theorem extra demands are the *geodesic and shear-free conditions.* The Ricci rotation coefficients are

$$\Gamma^a_{bc} = - \quad e^a_{\mu;\nu} e^\mu_b e^\nu_c. \qquad (41)$$

The e^3 direction will be geodesic if and only if $\Gamma_{424} = 0$, and it is shear free if and only if $\Gamma_{422} = 0$. The corresponding complex conjugate terms are $\Gamma_{414} = 0$ and $\Gamma_{411} = 0$. It was shown in DKS that $\Gamma_{42} = \Gamma_{42a}e^a = -dY - hY_{,4}\, e^4$. Therefore, the congruence e^3 is geodesic if $\Gamma_{424} = -Y_{,4}\,(1 - h) = 0$, and is shear free if $\Gamma_{422} = -Y_{,2} = 0$. Thus, the function $Y(x)$ defines a

shear-free and geodesic congruence iff $Y_{,2} = Y_{,4} = 0$, which means that dY is spanned by e^1 and e^3.
The equations $Y(x^\mu) = const.$ fixes the direction e^3, and together with $U(x^\mu) = const.$ it fixes one of the rays of the Kerr congruence (twistor, or geodesic line of a photon). From the complex point of view this null ray is a 'left' complex null plane (spanned by directions e^3 and e^1) [24, 23], The KS spacetimes are foliated on these null planes satisfying the extra condition of stationarity $\hat{K}Y = 0$.

Appendix B: Twistor variables and the Kerr Theorem

The function $U = x^\mu e^3_\mu$ sheds the light on geometrical meaning to the projective twistor coordinates λ^1 and λ^2. One sees that $\lambda^1 = x^\mu e^1_\mu$ is the 'first' tetrad components of a spacetime point x^μ, which in the null coordinates takes the form

$$\lambda^1 = x^n e^1_n = \partial_Y U = \zeta - Yv; \quad \lambda^2 = U - \bar{Y}\partial_Y U = u + Y\bar{\zeta}. \tag{42}$$

Since U is real, the complex conjugate combination,

$$\bar{\lambda}^1 = x^\mu e^2_\mu = \bar{\zeta} - \bar{Y}v = \partial_{\bar{Y}} U, \tag{43}$$

describes the right null plane. These relations may be completed by the fourth tetrad component $x^\mu e^4_\mu = v - hU$. Therefore, the twistor coordinates are

$$T^A = \{Y, \quad \zeta - Yv, \quad u + Y\bar{\zeta}\}. \tag{44}$$

The Kerr Theorem [24, 6, 18, 22, 23] determines the geodesic and shear-free Principal Null Congruences via an arbitrary analytic generating function F of the projective twistor variables $T^A = \{Y, \quad \zeta - Yv, \quad u + Y\bar{\zeta}\}$, as the solution $Y(x)$ of the equation $F(T^A) = 0$, with a subsequent use of the relation (7).
The Kerr congruences for a Kerr's source in a general position with an arbitrary finite boost corresponds to a quadratic in Y generating function F, [23, 24, 25, 26],

$$F = A(Y - Y^+)(Y - Y^-) = AY^2 + BY + C, \tag{45}$$

where the coefficients are given by the relations [25]

$$\begin{aligned} A &= (\bar{\zeta} - \bar{\zeta}_0)\dot{v}_0 - (v - v_0)\dot{\bar{\zeta}}_0; \\ B &= (u - u_0)\dot{v}_0 + (\zeta - \zeta_0)\dot{\bar{\zeta}}_0 - (\bar{\zeta} - \bar{\zeta}_0)\dot{\zeta}_0 - (v - v_0)\dot{u}_0; \\ C &= (\zeta - \zeta_0)\dot{u}_0 - (u - u_0)\dot{\zeta}_0. \end{aligned} \tag{46}$$

In the rest frame we use (35) and obtain

$$A = (x - iy)/2, \quad B = z + ia, \quad C = -(x + iy)/2\ . \tag{47}$$

The roots of the equation $F = 0$ are given by

$$Y^{\pm} = (-B \pm \Delta)/2A, \tag{48}$$

where $\Delta = (B^2 - 4AC)^{1/2}$, and we obtain from (33) that

$$\Delta = \sqrt{x^2 + y^2 + (z + ia)^2} \equiv \tilde{r} \tag{49}$$

is the complex radial distance, and two roots $Y^{\pm}$ of the eq. $F(Y) = 0$ are

$$Y^{\pm} = (-B \pm \tilde{r})/2A. \tag{50}$$

We have to match this expression with (8). For Y^+ we set $z = r\cos\theta$ and obtain the relations

$$e^{i\phi}\tan\frac{\theta}{2} = Y^+ = \frac{-B + r + ia\cos\theta}{2A} = \frac{-r\cos\theta - ia + r + ia\cos\theta}{x - iy}$$
$$= 2\,\frac{\sin^2\frac{\theta}{2}\,(r - ia)}{x - iy}\,,$$

which yield the coordinate relations

$$x - iy = (r - ia)e^{-i\phi}\sin\theta, \quad z = r\cos\theta \tag{51}$$

compatible with (9). Now, we would like to check the second root Y^-, retaining the relation $z = r\cos\theta$. We can do it by a transfer to the negative sheet, with the replacement $r \to -r$ and $\cos\theta \to -\cos\theta$. We obtain the relations $Y^- = (-B - r - ia\cos\theta)/2A = -2\cos^2\frac{\theta}{2}(r + ia)/(x - iy)$ which are compatible with $Y^- = -e^{i\phi}\cot\frac{\theta}{2}$ and the coordinate relations

$$x - iy = (r + ia)e^{-i\phi}\sin\theta, \quad z = r\cos\theta. \tag{52}$$

Antipodal map. We obtain that the second root Y^- is related with the replacement $r + ia\cos\theta \to -(r + ia\cos\theta)$ which is effectively a transfer to the negative sheet of the Kerr metric $r \to -r$ and the replacement $\cos\theta \to -\cos\theta$, accompanied by the antipodal transformation of the projective angular coordinate, $I_A* : Y^+ \to Y^-$,

$$Y^- = -\frac{1}{(Y^+)^*}. \tag{53}$$

Note, that the antipodal transformations of the null vector field $k^\mu = (k^0, \vec{k})$ leads to reversal of its time-direction $\acute{k}^\mu = I_A * k^\mu = (-k^0, \vec{k})$.
Complex expansion of the congruence $Z = (expansion) + i\ (twist)$ is determined in DKS as the tetrad derivative

$$Z = Y_{,1}\,, \tag{54}$$

and is related to the generating function F as follows

$$Z = -PF_Y^{-1}. \tag{55}$$

On the other hand, one sees that $F_Y \equiv d_Y F = 2YA + B = (x-iy)Y^{\pm} - z + ia = \tilde{r}$, and therefore, we have

$$\tilde{r} = F_Y = -P/Z, \tag{56}$$

and we obtain that the Kerr theorem determines almost all the necessary functions of the Kerr-Schild metrics, ([26]).

Null direction k in the Kerr angular variables. We start from (9) and (8) and obtain $\zeta = \frac{1}{\sqrt{2}}(x+iy) = \frac{1}{\sqrt{2}}(r+ia)e^{i\phi}\sin\theta$ which yields $d\zeta/\zeta = dr/(r+ia) + id\phi + \cos\theta d\theta/\sin\theta$. We note that

$$P = -\frac{1}{\sqrt{2}}(1+Y\bar{Y}) = -\frac{1}{\sqrt{2}\cos^2\frac{\theta}{2}} \tag{57}$$

and obtain $\bar{Y}d\zeta = -\frac{P}{2}(r+ia)[\sin^2\theta dr/(r+ia) + i\sin^2\theta d\phi + \sin\theta\cos\theta d\theta]$. We have also $du - Y\bar{Y}dv = P(dt - \cos\theta dz)$. As a result, for $k = e^3/P = \frac{1}{P}(du + \bar{Y}\zeta + Y\bar{\zeta} - Y\bar{Y}dv)$ we obtain

$$k = k_\mu dx^\mu = dt - dr + a\sin^2\theta d\phi, \tag{58}$$

which differs by sign from the eq.(7.5) of DKS. Note, that in DKS authors retain in sec.7 the notation e^3 for its normalized value $k_\mu dx^\mu = e^3/P$.

Appendix C: Retarded time and basic directional derivatives

Let us now calculate some important tetrad derivatives. The tetrad derivatives of Y are determined by definition of Z, (54): ($Z = Y,_1$); by the conditions of the Kerr theorem: the geodesic and shear-free conditions for PNC are $Y,_2 = Y,_4 = 0$; and by the following from the Kerr theorem relation for $Y,_3$. Therefore, for the geodesic and shear-free PNC we have:

$$Y,_1 = Z, \quad Y,_2 = Y,_4 = 0, \quad Y,_3 = -P_{\bar{Y}}Z/P. \tag{59}$$

Conjugate relations yield:

$$\bar{Y},_1 = \bar{Z}, \quad \bar{Y},_1 = \bar{Y},_4 = 0, \quad \bar{Y},_3 = -P_Y\bar{Z}/P. \tag{60}$$

We obtain now

$$dY = Y,_1 e^1 + Y,_3 e^3 = Z(e^1 - \frac{P_{\bar{Y}}}{P}e^3); \quad d\bar{Y} = \bar{Z}(e^2 - \frac{P_Y}{P}e^3), \tag{61}$$

which yields

$$dY \wedge d\bar{Y} = Z\bar{Z}(e^1 \wedge e^2 - \frac{P_Y}{P}e^1 \wedge e^3 + \frac{P_{\bar{Y}}}{P}e^2 \wedge e^3). \tag{62}$$

We obtain also

$$de^3 = Z(e^1 - \frac{P_{\bar{Y}}}{P} e^3) \wedge e^2 + \bar{Z}(e^2 - \frac{P_Y}{P} e^3) \wedge e^1 = dY \wedge e^2 + d\bar{Y} \wedge e^1. \quad (63)$$

One can easily obtain from (8) that $\cos\theta = \frac{1-Y\bar{Y}}{1+Y\bar{Y}}$; $\sin\theta = \frac{2|Y|}{1+Y\bar{Y}}$, and we obtain $(\cos\theta)_{,1} = -Z\bar{Y}/P^2$; $(\cos\theta)_{,2} = -\bar{Z}Y/P^2$; $(\cos\theta)_{,3} = -(Z + \bar{Z})\frac{(Y\bar{Y})^2}{\sqrt{2}P^3}$; $(\cos\theta)_{,4} = 0$. Taking into account (59), we obtain

$$\begin{aligned} &P_{,1} = -2^{-1/2}\bar{Y}Z; \quad P_{,2} = -2^{-1/2}Y\bar{Z}\ ; \\ &P_{,3} = -P_Y P_{\bar{Y}}(Z+\bar{Z})/P; \quad P_{,4} = 0. \end{aligned} \quad (64)$$

From (56) we derive $\tilde{r}_{,2} = -P_{,2}/Z + PZ_{,2}/Z^2$, and using the given in [2] commutation relation

$$Z_{,2} = (Z - \bar{Z})Y_{,3}\,, \quad (65)$$

we obtain $\tilde{r}_{,2} = -P_{\bar{Y}}$. For the Kerr metric at rest we have $\tilde{r}_{,2} = r_{,2} + ia(\cos\theta)_{,2}$, and since $(\cos\theta)_{,2} = -\bar{Z}Y/P^2$ we obtain $r_{,2} = -P_{\bar{Y}} + ia\bar{Z}Y/P^2$. So far as r is real the complex conjugation yields $r_{,1} = -P_Y - iaZ\bar{Y}/P^2$, and finally $\tilde{r}_{,1} = r_{,1} + ia(\cos\theta)_{,1} = -P_Y - 2iaZ\bar{Y}/P^2$ The DKS relation $Z_{,4} = -Z^2$, yields $\tilde{r}_{,4} = -(P/Z)_{,4} = -P$. To get derivative $\tilde{r}_{,3} = -P_{,3}/Z + PZ_{,3}/Z^2$ we use the given in DKS expression $Z_{,3} = Y_{,31} + hZ^2 - Y_{,3}\bar{Y}_{,3}$. For the considered solution at rest it gives $Z_{,3} = -2ia\bar{Y}P_{\bar{Y}}Z^3/P^4 - Z^2(PP_{Y\bar{Y}} - P_Y P_{\bar{Y}})/P^2 + hZ^2 - P_Y P_{\bar{Y}} Z\bar{Z}/P^2$, which yields $\tilde{r}_{,3} = 2P_Y P_{\bar{Y}}/P - P_{Y\bar{Y}} + hP - 2ia\bar{Y}P_{\bar{Y}}Z/P^3$. Summarizing, we obtain

$$\begin{aligned} &\tilde{r}_{,1} = -P_Y - 2ia\bar{Y}Z/P^2; \quad \tilde{r}_{,2} = -P_{\bar{Y}}; \\ &\tilde{r}_{,3} = 2P_Y P_{\bar{Y}}/P - P_{Y\bar{Y}} + hP - 2ia\bar{Y}P_{\bar{Y}}Z/P^3; \quad \tilde{r}_{,4} = -P; \end{aligned} \quad (66)$$

It should be noted that these relations are valid only for the Kerr metric at rest. Let us also summarize all the derivations $Z_{,a}$:

$$\begin{aligned} &Z_{,1} = 2ia\bar{Y}Z^3/P^3, \quad Z_{,2} = (Z-\bar{Z})Y_{,3}\,, \\ &Z_{,3} = -2ia\bar{Y}P_{\bar{Y}}Z^3/P^4 - Z^2(PP_{Y\bar{Y}} - P_Y P_{\bar{Y}})/P^2 + \\ &\qquad hZ^2 - P_Y P_{\bar{Y}}Z\bar{Z}/P^2, \qquad Z_{,4} = -Z^2. \end{aligned} \quad (67)$$

We are approaching to the our aim to consider the properties of our retarded time parameter $\tau = t - \tilde{r}$. Since $t = 2^{-1/2}(v - u)$, one can easily obtain

$$\begin{aligned} &t_{,1} = -P_Y, \quad t_{,2} = -P_{\bar{Y}}, \\ &t_{,3} = P_{Y\bar{Y}} + hP, \quad t_{,4} = -P, \end{aligned} \quad (68)$$

and therefore, for the retarded time $\tau = t - \tilde{r}$ we have

$$\tau_{,1} = t_{,1} - \tilde{r}_{,1} = -2ia\bar{Y}Z/P^2, \quad (69)$$

$$\tau_{,2} = 0, \quad (70)$$

$$\tau_{,3} = \frac{1}{P} + 2iaZ\bar{Y}P_{\bar{Y}}/P^3 \quad (71)$$

$$\tau_{,4} = 0. \quad (72)$$

It yields

$$d\tau = \tau,_1 e^1 + \tau,_3 e^3 = \frac{1}{P} e^3 + \tau,_1 Z^{-1} dY, \tag{73}$$

and

$$d\tau \wedge dY = \frac{1}{P} e^3 \wedge dY. \tag{74}$$

Finally, using these relations, one can check action of the operator $\mathcal{D} = \partial_3 - Z^{-1} Y,_3 \partial,_1 - \bar{Z}^{-1} \bar{Y},_3 - \partial,_2$ on the retarded time τ, and we obtain

$$\mathcal{D}\tau = \tau,_3 + (P_{\bar{Y}}/P)\tau,_1 = \frac{1}{P}. \tag{75}$$

Appendix D. Complex vector potential

For the complex vector potential (27), where $\bar{\chi} = \int A d\bar{Y}$, and Y and τ being kept constant, we have to prove that the complex components of the electromagnetic field strength (24) are obtained from the relation

$$d\mathcal{A} = \mathcal{F}_{12} e^1 \wedge e^2 + \mathcal{F}_{31} e^3 \wedge e^1 + \mathcal{F}_{34} e^3 \wedge e^4, \tag{76}$$

where $\mathcal{F}_{12} = \mathcal{F}_{34} = AZ^2$ and $\mathcal{F}_{31} = \gamma Z - (AZ),_1$. This fact is not trivial since the analogous relation obtained in DKS cannot be separated onto complex conjugate components. The calculations are rather cumbersome. We use the auxiliary relations

$$dY \wedge d\bar{Y} = Z\bar{Z} e^1 \wedge e^2 + Z\bar{Y},_3 e^1 \wedge e^3 - \bar{Z} Y,_3 e^2 \wedge e^3; \tag{77}$$

and

$$de^3 = (Z - \bar{Z}) e^1 \wedge e^2 - Y,_3 e^2 \wedge e^3 - \bar{Y},_3 e^1 \wedge e^3. \tag{78}$$

We have $d\mathcal{A} = (AZ),_1 e^1 \wedge e^3 + A,_2 Z e^2 \wedge e^3 + AZ,_2 e^2 \wedge e^3 + AZ,_4 e^4 \wedge e^3 + AZ de^3 - A d\bar{Y} \wedge dY - (\int \dot{A} d\bar{Y}\ ) \, d\tau \wedge dY$. To develop this expression we use $A,_2 Z = 2\bar{Z} A Y,_3$, (DKS eq.(5.42)), and also $Z,_2 = (Z - \bar{Z}) Y,_3$, $Z,_4 = -Z^2$ and (78). Further, we replace $\dot{A}$ by the second EM field equation $\dot{A} = -(P\gamma),_{\bar{Y}}$, obtained in [14, 5], which allows us to integrate the last term. Finally, we use (77),(74) and $dY = Z e^1 + Y,_3 e^3$, and obtain

$$d\mathcal{A} = [(AZ),_1 - \gamma Z] e^1 \wedge e^3 + AZ^2 e^3 \wedge e^4 + AZ^2 e^1 \wedge e^2 \ , \tag{79}$$

which corresponds to (24) for autodual components of the EM strength.

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