Causal Propagation in Noncommutative Field Theory*

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Abstract

Operatorial methods offer an insightful alternative to the usual Wigner-Weyl-Moyal approach to noncommutative field theories. In particular one can prove easily that the elementary degrees of freedom are bilocal, and live on a reduced configuration space. Causality issues become particularly transparent in this framework. Dispersion relations can also be formulated in a quite standard way.

1. Introduction

Consider a scalar field ϕ defined over an operatorial space-time,

$$\phi = \phi(\hat{x}^\mu), \qquad [\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu}(\hat{x}), \tag{1}$$

with $x^\mu \in \{x^0 = t, x^1 = x, x^2 = y, x^3 = z\}$. For technical reasons, let us restrict to a commutative time t, a commutative z-axis, and a Heisenberg type noncommutativity

$$[\hat{x}, \hat{y}] = i\theta\hat{I}. \tag{2}$$

$\hat{I}$ is the identity operator; θ is a constant having the dimension of an area.

One should wonder why such a theory is interesting. Historically, Heisenberg hoped that such noncommutativity may smooth out divergences in quantum field theory. Through Pauli and Oppenheimer this idea reached Snyder, who wrote the first paper on the subject [1]. The hope of Heisenberg was not borne out, however, in spite of interesting developments. Another reason, which will emerge in what follows, is that this theory is nonlocal, apparently the first nonlocal theory that can be kept under relatively good control. However, causality is usually an issue in nonlocal theories. It will be one of our main concerns.

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Nonrelativistically, causality simply means that some cause-effect ordering is respected. Relativistically, one must add the condition that any velocity related to some physical propagation should be at most the velocity of light, $v \leq c$. For a quantum field ϕ depending on $x^\mu = (x^0 = t, x^1, x^2, x^3)$ this formulation is not very useful however. One takes as an axiom the so-called micro-causality condition

$$[\phi(x), \phi(0)] = 0, \qquad t^2 - \vec{x}^2 \leq 0. \tag{3}$$

It reflects the assumption that two events having space-like separation cannot influence each other.

Causality of noncommutative field theory (NCFT) is discussed in less than 1% of the literature of the subject, and even then with contradictory results [2, 3],[6, 7, 8], [9, 10, 11, 12].

The reason for the confusion is the use of the Weyl-Moyal quantization procedure, in which NC space is mapped to a continuum of same dimensionality, parameterized by the so-called Weyl symbols. By necessity, a "point" in Weyl symbol space has no precise correspondent in the physical (NC) space. The product of functions gets deformed to the Moyal star product

$$f(x) \cdot g(x) \quad \rightarrow \quad f(x) \star g(x) \equiv \lim_{y \rightarrow x} \exp\left(\frac{i}{2}\theta^{\mu\nu}\partial^x_\mu \partial^y_\nu\right) f(x)g(y). \tag{4}$$

Two ambiguities appear while extending (3) to NC fields:

1. Is the commutator or the star-commutator [2] appropriate in (3)?

2. What is a "space-like interval" when coordinates can not be sharply measured simultaneously? Several conditions were used in the literature, for events separated by the quadri-vector $(\Delta t, \Delta\vec{r})$ *in Weyl space*:

a) usual light-cone [9]: $\Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2 \leq 0$ - too strong.

b) "light-wedge" condition: $\Delta t^2 - \Delta z^2 \leq 0$ [6, 7] - too weak. If no z is available, then one returns to nonrelativistic dynamics...

c) intermediate: $\Delta t^2 - \Delta z^2 \leq 2\theta$ - still inappropriate [8].

Our proposal will be to drop *one* of the two noncommuting coordinates (say y), and then require zero commutator provided $\Delta t^2 - \Delta x^2 - \Delta z^2 \leq 0$, but in *physical space*, not in Weyl space. We will show - disproving previous claims, that NC theories with commuting time *are* causal.

2. Dimensional Reduction and Bilocality

Consider a $(2+1)$-dimensional scalar $\Phi(t, \hat{x}, \hat{y})$, defined over a commuting time t and a pair of NC coordinates satisfying $[\hat{x}, \hat{y}] = i\theta\hat{I}$. The operators $\hat{x}$ and $\hat{y}$ act on a harmonic oscillator Hilbert space $\mathcal{H}$ in the usual way. Choose here the basis $\{|x>\}$ of eigenstates of $\hat{x}$: $\hat{x}|x>= x|x>$, $\hat{y}|x>= -i\theta\frac{\partial}{\partial x}|x>$.

To quantize Φ [4], start with a usual classical commuting field, expanded into normal modes with coefficients a and a^*. Upon usual field quantization, a and a^* become operators acting on a standard Fock space $\mathcal{F}$. To introduce NC space, apply the Weyl (not Weyl-Moyal!) quantization procedure to the exponentials $e^{i(k_x x+k_y y)}$. The result is

$$\Phi = \int\int \frac{dk_x dk_y}{2\pi\sqrt{2\omega_{\vec{k}}}} \left[\hat{a}_{k_x k_y} e^{i(\omega_{\vec{k}} t - k_x\hat{x} - k_y\hat{y})} + \hat{a}^\dagger_{k_x k_y} e^{-i(\omega_{\vec{k}} t - k_x\hat{x} - k_y\hat{y})}\right]. \quad (5)$$

Φ acts on a direct product of two Hilbert spaces, $\Phi : \mathcal{F}\otimes\mathcal{H} \to \mathcal{F}\otimes\mathcal{H}$. It creates (destroys) excitations represented by "operatorial plane wave" $e^{i(\omega_{\vec{k}} t - k_x\hat{x} - k_y\hat{y})}$.

It is simpler to saturate the action of Φ on $\mathcal{H}$ by working with expectation values $< x'|\Phi|x >: \mathcal{F} \to \mathcal{F}$. *Bilocality appears explicitly* due to:

$$< x'|e^{i(k_x\hat{x}+k_y\hat{y}}|x >= e^{ik_x(x+k_y\theta/2)}\delta(x'-x-k_y\theta) = e^{ik_x\frac{x+x'}{2}}\delta(x'-x-k_y\theta). \quad (6)$$

The span along the x axis is $(x'-x) = \theta k_y$, and the energy is [4]

$$\omega_{\vec{k}} = \sqrt{k_x^2 + \frac{\Delta x^2}{\theta^2} + m^2}. \quad (7)$$

One notices the intrinsic IR/UV-dual character of the dipoles: both big momentum (UV) and big extension (IR) give big energy. The second term reminds a string stretched between two separated D-branes. Finally,

$$< x'|\Phi|x >= \int \frac{dk_x}{2\pi\sqrt{2\omega_{k_x,k_y}}}\left[\hat{a}_{k_x,k_y} e^{i(\omega_{\vec{k}} t - k_x\frac{x+x'}{2})} + \hat{a}^\dagger_{k_x,-k_y} e^{-i(\omega_{\vec{k}} t + k_x\frac{x+x'}{2})}\right] \quad (8)$$

with $k_y = (x'-x)/\theta$. Thus, $< x'|\Phi|x >$ annihilates a linear combination of rods of (arbitrary) momentum k_x and (fixed) length θk_y, and creates rods of momentum k_x and length $-\theta k_y$. It is not anymore a local operator, in contrast to usual field theory. Failure to recognize that feature explicitly in the Moyal formulation may lead to erroneous conclusions about causality (see [3] however).

3. Perturbation Theory

Correlators

Two-point correlation functions for dipoles are built out of

$$\langle 0|< x_4|\Phi|x_3><x_2|\Phi|x_1>|0\rangle = \int \frac{dk_x}{8\pi^2\omega_{\vec{k}}} e^{ik_x[\frac{x_3+x_4}{2}-\frac{x_1+x_2}{2}]}\delta(x_4-x_3-x_2+x_1). \quad (9)$$

Above, $k_y = (x' - x)/\theta$ is fixed, $\omega_{\vec{k}} = \omega_{k_x,k_y}$ obeys (7). Eq. (9) differs from usual (1 + 1)-correlators, $\langle 0|\phi(X_2)\phi(X_1)|0\rangle$, with $X_1 = (x_1 + x_2)/2$ and $X_2 = (x_3 + x_4)/2$, through a. the $\frac{(x'-x)^2}{\theta^2}$ term in (7), b. the delta function $\delta([x_4 - x_3] - [x_2 - x_1])$, which ensures that the length of the rod is conserved.

Interactions

The action is

$$S = \frac{1}{2}\int \mathrm{d}t\,\mathrm{Tr}_{\mathcal{H}}\left[\dot{\Phi}^2 - (\partial_x\Phi)^2 - (\partial_y\Phi)^2 - m^2\Phi^2 - 2V(\Phi)\right]. \qquad (10)$$

We will exemplify with a quartic potential, $V(\Phi) = \frac{g}{4!}\Phi^4$, which can be written in our bilocal notation as

$$\int dtTr_{\mathcal{H}}V(\Phi) = \frac{g}{4!}\int dt\int_{x,a,b,c} < x|\Phi|a >< a|\Phi|b >< b|\Phi|c >< c|\Phi|x > . \qquad (11)$$

The basic 'vertex' for four-dipole scattering follows from

$$\langle -\vec{k}_3, -\vec{k}_4| : \int dt\int_{x,a,b,c} < x|\Phi|a >< a|\Phi|b >< b|\Phi|c >< c|\Phi|x >: |\vec{k}_1, \vec{k}_2\rangle. \qquad (12)$$

The momenta $\vec{k}_{i,i=1,2,3,4}$ each have two components: $\vec{k}_i = (k_i, l_i)$. k_i is the momentum along x, whereas l_i represents the dipole extension along x. One obtains the conservation laws $k_1 + k_2 + k_3 + k_4 = 0$ and $l_1 + l_2 + l_3 + l_4 = 0$. The final result differs from the four-point scattering vertex of (2 + 1) commutative particles with momenta $\vec{k}_i = (k_i, l_i)$ only through the phase

$$e^{-\frac{i\theta}{2}\sum_{i<j}(k_i l_j - l_i k_j)}. \qquad (13)$$

This is precisely the star-product modification of the usual Feynman rules. The phase (13) appears due to the bilocal nature of generic $< x'|\Phi|x >$'s.

One has to integrate over both the momentum and length of the dipole circulating in a loop. This $\frac{1}{2\pi}\int dk_{loop}\int dl_{loop}$ integration, together with the dispersion relation (7), brings back into play - especially as far as divergences are concerned - the y direction.

IR/UV

The momentum in the conjugate (y) direction became the lenght (Δx) of the dipole.

Consider (4 + 1)-dimensions, $t, \hat{x}, \hat{y}, \hat{z}, \hat{u}$, with $[\hat{x}, \hat{y}] = [\hat{z}, \hat{w}] = i\theta$. In the $\{|x, z >\}$ basis, one has a commutative space spanned by the axes x and

z, on which dipoles with momentum $\vec{p} = (p_x, p_z)$ and length $\vec{l} = (l_x, l_z) = \theta(p_y, p_w)$ evolve. During the scattering, four such dipoles meet in a four-edged polygon of area $\mathcal{A}$ (figure 1a).

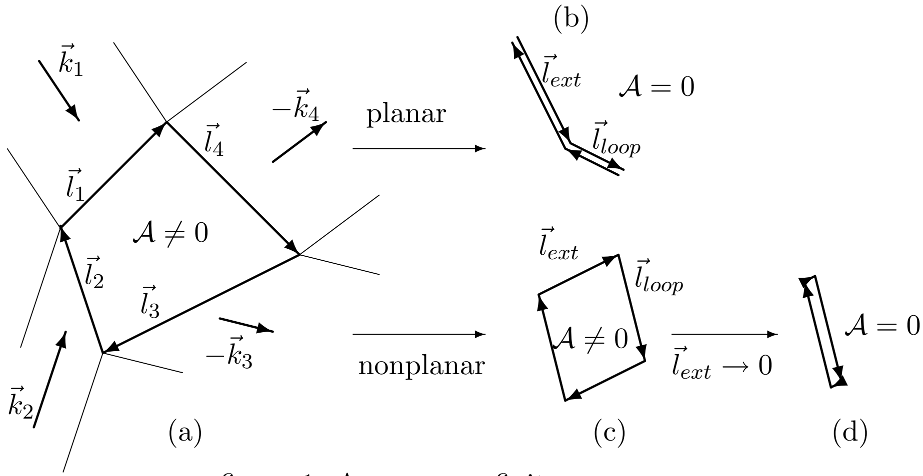

figure 1: Area versus finiteness

The one-loop correction to the propagator involves both planar and non-planar diagrams, as follows.

Planar case: adjacent dipole fields are contracted. Momentum and length conservation enforce the polygon to degenerate into a one-dimensional, zero-area object (figure 1b). UV divergences persist.

Nonplanar case: due to the nonadjacent contraction the area $\mathcal{A}$ does not go to zero (figure 1c) unless the external dipole length vanishes (figure 1d). $\mathcal{A} \neq 0$ appears thus to be related to the disappearance of UV divergences. Actually, the true regulator is the phase (13), which is ineffective when $\mathcal{A} = 0$ in *both* the $|x, z>$ and $|y, u>$ bases. That corresponds to zero external length *and* momentum in the dipole picture, which means that the resulting divergence is half IR ($\vec{p}_{ext} = 0$) and half UV ($\vec{l}_{ext} = 0$)! In Weyl space this is just the usual zero external momentum, say $p_\mu^{ext} = 0$, and one speaks about an IR divergence. NCFT is somehow in between usual FT and string theory: when the interaction vertex is a point, UV infinities appear; when it opens up, amplitudes are finite.

4. Symmetries

Introduce the representation of the previous sections in the equations of motion, and reintroduce the commutative z direction. We use the notation

$$< x'|\phi|x > \equiv \phi(x', x) \equiv \phi(\bar{x}, \Delta x), \quad \bar{x} \equiv \frac{x + x'}{2}, \quad \Delta x \equiv (x' - x) \qquad (14)$$

The free equations of motion follow from the action

$$S = Tr_H \int dt \int dz \left((\dot{\phi})^2 + \frac{1}{\theta^2}[\hat{x},\phi]^2 + \frac{1}{\theta^2}[\hat{y},\phi]^2 - (\partial_z\phi)^2 + m^2\phi^2 \right), \quad (15)$$

and are written operatorially for $\phi(t, \hat{x}, \hat{y}, z)$ as

$$(\partial_t^2 - \partial_z^2 + m^2)\phi + \frac{1}{\theta^2}[\hat{y},[\hat{y},\phi]] + \frac{1}{\theta^2}[\hat{x},[\hat{x},\phi]] = 0, \quad (16)$$

which is nothing else than an operatorial wave equation, given that (2) implies

$$[\hat{x}, \phi(\hat{x},\hat{y})] = i\theta \frac{\partial \phi}{\partial \hat{y}} \qquad [\hat{y}, \phi(\hat{x},\hat{y})] = -i\theta \frac{\partial \phi}{\partial \hat{x}}. \quad (17)$$

Sandwiching the operatorial equation between $|x>$ states, one gets rid of noncommutativity and obtains the wave equation

$$\left(\partial_t^2 - \partial_{\bar{x}}^2 - \partial_z^2 + \frac{(x'-x)^2}{\theta^2} + m^2 \right) \phi(x,x') = 0. \quad (18)$$

for a dipole living in (2+1) commutative dimensions at $t, \bar{x}, z$ *and* having extension Δx. We notice the full agreement with the dispersion relation (7). Eq. (18) follows from $< x'|[\hat{x},[\hat{x},\phi]]|x >= (x'-x)^2\phi$ and from $< x'|[\hat{y},[\hat{y},\phi]]|x >= -\partial_{\bar{x}}\phi(\bar{x},\Delta x)$ [to obtain the latter, operate with the commutator on $\phi(\hat{x},\hat{y})$, Fourier expand, sandwich between $|x>$ bras and kets]. In the interacting case, the relevant Lagrangian is thus

$$2L = (\partial_t\phi)^2 - (\partial_{\bar{x}}\phi)^2 + [(\theta^{-1}\Delta x)^2 + m^2]\phi^2 - 2V(\phi) \quad (19)$$

and includes the potential $V(\phi)$ for the fields, for instance $V = \lambda\phi^4$. The Lagrangian L has the property of being invariant under Lorentz boosts along the $\bar{x}$-axis, as well as along the z-axis, independently. The only think to prove in this respect is the invariance of the third term in the RHS. This immediately follows from the tensorial character of $\theta = \theta_{xy} \sim xy$ and the usual Lorentz transformation of Δx. These symmetries, which we found in the bilocal representation for NC space, are at variance with the claims usually made within the Moyal approach, that the symmetry group is the product between the rotation group $O(2)$ in the $x - y$ NC space and the Lorentz group $O(1,1)$ in $t - z$ space.

5. Causality

Free NC fields behave like lower-dimensional commutative fields. A free (1+1)-dimensional dipole [resulting from a 2+1 NC theory] with endpoints situated at x and x' behaves like a commutative $(1+1)$ point particle centered at $\frac{x+x'}{2}$, but with a modified dispersion relation $\omega^2 = k_x^2 + \frac{(x-x')^2)}{\theta^2}$.

In conclusion causality is demonstrated in the same way as for free fields. Free NC field theories are thus causal, contrary to previous claims [2].

For interacting fields, one expects the dipolar character of the degrees of freedom to manifest, as e.g. in perturbation theory [4]. It is however remarkable that as far as the micro-causality condition is concerned, bilocality has little influence. For, consider the vanishing of the commutator to hold,

$$[\phi(t_1, \bar{x}_1), \phi(t_2, \bar{x}_2)] = 0 \tag{20}$$

with $\bar{x}_1 = \frac{x_1 + x_1'}{2}$, $\bar{x}_2 = \frac{x_2 + x_2'}{2}$ being the average positions (centers of mass) of the two dipoles considered. We want (20) to be true when the interval, defined with respect to the average dipole positions, is space-like,

$$(t_1 - t_2)^2 - (\bar{x}_1 - \bar{x}_2)^2 \leq 0. \tag{21}$$

Equations (20, 21) are however generically equivalent to

$$[\phi(t, \bar{x}), \phi(t, \bar{y})] = 0, \qquad \vec{x} \neq \vec{y}, \tag{22}$$

provided one can apply a boost along x to render equal the two times appearing in Eq. (20).

This requires the $(1+1)$-dimensional dipole theory to be invariant under boosts in the x-direction, which we already demonstrated. (a fact completely overlooked in the literature, which claims that the only invariance left after NC is imposed is a product of $O(2)$ for the NC part and of the Lorentz group, e.g. $SO(1,1)$, for the rest). In consequence, Eqs. (20, 21) are tantamount, via a boost, to

$$e^{iH't}[\phi(0, \bar{x}), \phi(0, \bar{y})]e^{-iH't} = 0 \tag{23}$$

which is true at $t = 0$, since this is by definition the time at which the fields behave like free fields (H' denotes the interacting part of the Hamiltonian, including V, in the interaction representation).

Adding now the (passive) commutative coordinate z, we conclude that the correct causality criterion for NCFT is

$$[\phi(t_1, \bar{x}_1, z_1), \phi(t_2, \bar{x}_2, z_2)] = 0, \quad (t_1 - t_2)^2 - (\bar{x}_1 - \bar{x}_2)^2 - (z_1 - z_2)^2 \leq 0. \tag{24}$$

6. Dispersion Relations

Classical

A signal $A(t)$ is zero for $t < 0$ provided its Fourier transform $a(\omega)$ is analytic in the upper half-plane (for $Im\omega \geq 0$). For light propagation in one dimension, t is replaced by $\tau \equiv t - z/c$. The above applies to (linear) transfer functions in general.

If one goes to three dimensions and looks at propagation at nonzero angles with respect to the propagation axis (z), the situation is more complicated but will not concern us here. For massive particles ($v < c$) an impediment also appears: one has to use frequencies $\omega \in (-m, m)$, hence in the unphysical region, to form sharp wave packets.

Dipoles

In NCFT, the most one can hope for a localized quanta is a dipole with end-points localized at x and x' on the x-axis, with center-of-mass at $\bar{x} = \frac{x+x'}{2}$ moving on the trajectory $x_0(t)$. If the dipole length is fixed to be $x' - x \equiv \Delta x = l > 0$, the excitation has the form

$$\delta(\frac{x+x'}{2} - x_0(t))\delta(x - x' - l) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} dk_x e^{[ik_x\bar{x} - x_0(t)]}\delta(\Delta x - l) \quad (25)$$

(alternatively, integration over the dipole size $\Delta x = k_y\theta$ is possible). Consider now the scattering in one-space dimension of such dipoles via an obstacle sitting at $x = 0$ (perfect localization along x means the scatterer has no momentum along y, otherwise a width in x would appear for it too). We let approach from the left an incident bipolar excitation of the type (25), at constant velocity,

$$x_0(t) = vt, \qquad v = \frac{\omega}{k_x}. \quad (26)$$

The resulting field, incident plus scatterer, will be of the forma

$$F_{inc}(x, x', t) + F_{scatt}(r, r', t) \sim \delta(\bar{x} - \omega t/k)\delta(\Delta x - l) + \int d\omega f(\omega)e^{i(k\bar{r} - \omega t)}\delta(\Delta r - L). \quad (27)$$

$L \equiv r' - r$ denotes the dipolar length of the scattered wavelets, with dipole end-points r and r' and center of mass at $\bar{r}$. Given that we are in one-dimension, no $\frac{1}{r}$ factor is present, and no angular dependence appears in the scattering amplitude $f(\omega)$, which is a function only of the frequency and characterizes the scatterer. Causality will provide constraints on f in much the same way as is usual, with minor complications described below.

The center-of-mass of the incident wave-packet arrives (from the left) at the origin $x = 0$ at time $t = 0$. This means its advanced end-point, x', reaches the origin at $-\Delta t = -\frac{l}{2v}$. By way of causality, there will be no scattered signal before that moment. Admitting that the scattered signal propagates with at most the speed of light, we will in general have

$$F_{scatt}(r, r', t) = 0 \qquad if \qquad r' \geq c(t + \Delta t), \quad (28)$$

or ($v_2 = v_1$ for localized target)

$$F_{scatt}(r, r', t) = \int d\omega f(\omega)e^{-i\omega(t - r'/v)}e^{-ik\delta r/2}\delta(\Delta r - L) = 0 \quad (29)$$

for

$$\tau \equiv t - \frac{r'}{c} \leq -\Delta t. \tag{30}$$

This implies that

$$e^{-ik\delta r/2}\delta(\Delta r - L)f(\omega) = \int_{-\Delta t}^{\infty} F(r, r', \tau) \tag{31}$$

is analytic in the upper complex ω-plane, on account of the finite inferior limit in the Fourier integral (the new fact here that this limit is non-zero is irrelevant to the dispersion relation standard derivation arguments).

Analyticity via LSZ

The derivation of dispersion relations through the LSZ formalism seems insensitive to NC [12] (see also [10]). For an opposite opinion, see [11]. Starting with the usual assumptions, $|\vec{k}, in >= |\vec{k}, out >$ for one-particle states, $< 0|j(x)|\vec{k} >= 0$ for the current $j(x) \equiv (\partial_t^2 - \nabla^2 + m^2)\Phi$, we define

$$S_{fi} \equiv < f, out|i, in >=< f, in|S|i, in > .$$

The retarded commutator reads $R(A(x), B(y)) = \theta(x_0 - y_0)[A(x), B(y)]$. The free field expansion stays the same,

$$\Phi_{in}(x) = \frac{1}{(2\pi)^3}\int \frac{d^3\vec{k}}{\sqrt{2\omega_{\vec{k}}}}(\hat{a}_{\vec{k}}e^{-ikx} + H.C.),$$

and only the expression for $\hat{a}_{\vec{k}}$ changes slightly,

$$\hat{a}_{\vec{k}} = i(2\pi)^{3/2}\int_z Tr_H \frac{e^{ikx}}{\sqrt{2\omega_{\vec{k}}}}\hat{\partial}_0\Phi(x).$$

For forward scattering ($p\prime = p, q\prime = q, k = p+q$), $S_{fi} = \delta_{fi} - 2\pi i\delta(p_f - p_i)T_{fi}$,

$$T_{fi} \overset{LSZ}{\sim} \int d^4w e^{ikw} < p|R[j(w/2)j(-w/2)]|p > \tag{32}$$

is an analytic function of k_0 in the upper half-plane, although expressions are less explicit, due to lack of spherical symmetry.

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