Enveloping algebra Noncommutative SM: Renormalisability and High Energy Physics Phenomenology *

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Abstract

In this talk we discuss enveloping algebra based noncommutative gauge field theory, constructed at the first order in noncommutative parameter θ, as an effective, anomaly free theory, with one-loop renormalizable gauge sector. Limits on the scale of noncommutativity parameter $\Lambda_{\rm NC}$, via related phenomenology and associated experiments, are analyzed and a firm bound to the scale of the noncommutativity is set around few TeV's.

The Standard Model (SM) of particle physics and the theory of gravity describe very well, as far as we know today, all physical phenomena from cosmological processes to the properties of subnuclear structures. Nevertheless, at extreme energy and/or very short distances -at the Planck scale- this theories fail to be compatible, which motivates the study of modified or alternative space-time structures that could help to solve the above mentioned difficulties or at least shed some light on them. These modified space-time structures arise in such frameworks as the quantized coordinates in string theory or in deformation quantization. The idea of noncommutative (NC) space-time, which can be realised in both of the above settings, has recently found more and more interest. In this paper we deal with noncommutative theories defined by means of the enveloping algebra approach, which allows to define gauge theories with arbitrary gauge groups, in particular that of the Standard Model. The research on these theories so far has successfully dealt with both theoretical and phenomenological aspects, which allow the confrontation of the theory with experiments.

One of the first example where noncommutativity (NC) was introduced is well known Heisenberg algebra. Motivations to construct models on

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noncommutative space-time are comming from: String Theory, Quantum Gravity, Lorentz invariance breaking, and by its own right. The star product ($\star$) definition is as usual. The $\star$-commutator and Moyal-Weyl $\star$-product of two functions are:

$$
\begin{aligned}
[x^\mu \stackrel{\star}{,} x^\nu] &= x^\mu \star x^\nu - x^\nu \star x^\mu = ih\theta^{\mu\nu}, & (1)\\
(f \star g)(x) &= e^{-\frac{i}{2}\theta^{\mu\nu}\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial y^\nu}} f(x)g(y)|_{y\to x}\,. & (2)
\end{aligned}
$$

Here θ is constant, antisymmetric and real 4×4 matrix; $h = 1/\Lambda_{\rm NC}^2$ is noncommutative deformation parameter. Symmetry in our model [1], using Seiberg-Witten map (SW) [2] is extended to enveloping algebra [1, 3]. Any enveloping algebra based model is essentialy double expansion in power series in θ [1, 3, 4, 5, 6]. In principle SW map express noncommutative functionals (parameters and functions of fields) spanned on the noncommutative space as a local functionals spanned on commutative space.

To obtain the action we first do the Seiberg-Witten expansion of NC fields in terms of commutative ones and second we expand the $\star$-product. This procedure generets tower of new vertices, however it is valid for any gauge group and arbitrary matter representation. Also there is no charge quantization problem and no UV/IR mixing [7]. Unitarity is satisfied for $\theta^{0i}=0$ and $\theta^{ij}\neq 0$ [8, 9]; however careful canonical quantization produces always unitary theory. By covariant generalization of the condition $\theta^{0i}=0$ to:

$$
\theta_{\mu\nu}\theta^{\mu\nu} \;=\; -\theta^2 = \frac{2}{\Lambda_{\rm NC}^4}\left(\vec{B}_\theta^2 - \vec{E}_\theta^2\right) > 0\;, \qquad (3)
$$

which is known as *perturbative unitarity condition* [10], there is no difficulties with unitarity in NC gauge theories. Finally covariant noncommutative Higgs and Yukawa couplings were constructed in [4].

There are two essential points in which NC gauge field theory (NCGFT) differ from standard model (SM) gauge theories. The breakdown of Lorentz invariance with respect to a fixed nonzero background field $\theta^{\mu\nu}$ (which fixes preferred directions) and the appearance of new interactions and the modification of standard ones. For example, triple–neutral–gauge boson, two fermion–two gauge bosons, direct photon-neutrino couplings, etc. Both properties have a common origin and appear in a number of phenomena at very high energies and/or very short distances.

In this article we consider θ-expanded theories, constructed as an effective, anomaly free [11] and one-loop renormalizable NCGFT [12, 14, 15, 16], at the first order in noncommutative parameter θ. Finally we discuss related phenomenology and determine the scale of noncommutativity $\Lambda_{\rm NC}$, [18, 19, 20].

Properties of θ-expanded noncommutative gauge field theory satisfie:

Covariant coordinates $\hat{x}^\mu = x^\mu + h\theta^{\mu\nu}\widehat{A}_\nu$ were in noncommutative theory introduced in analogy to covariant derivatives in ordinary theory.

Noncommutative gauge transformation, i.e. consider infinitesimal noncommutative local gauge transformation $\hat{\delta}$ of a fundamental matter field that carries a representation ρ_Ψ, which is in Abelian case fixed by the hypercharge, $\hat{\delta}\widehat{\Psi} = i\rho_\Psi(\widehat{\Lambda}) \star \widehat{\Psi}$.

Locality of the theory, i.e. star-product of two ordinary functions $f(x)$ and $g(x)$, determined by a Poisson tensor $\theta^{\mu\nu}$ and written in the form of expansion, is local function of f and g with finite number of derivatives at each order in θ.

Gauge equivalence for the theory, i.e. ordinary gauge transformations $\delta A_\mu = \partial_\mu\Lambda + i[\Lambda, A_\mu]$ and $\delta\Psi = i\Lambda \cdot \Psi$ induce noncommutative gauge transformations of the NC gauge and fermion fields $\widehat{A}$, $\widehat{\Psi}$ with NC gauge parameter $\widehat{\Lambda}$: $\delta\widehat{A}_\mu = \hat{\delta}\widehat{A}_\mu$ and $\delta\widehat{\Psi} = \hat{\delta}\widehat{\Psi}$.

Consistency condition require that any pair of noncommutative gauge parameters $\widehat{\Lambda}$, $\widehat{\Lambda'}$ satisfy

$$\widehat{[\Lambda, \Lambda']} = [\widehat{\Lambda}, \widehat{\Lambda'}] + i\delta_\Lambda\widehat{\Lambda'} - i\delta_{\Lambda'}\widehat{\Lambda}\,. \tag{4}$$

Enveloping algebra-valued noncommutative gauge parameters and fields, i.e. for the enveloping algebra-valued gauge transformation, the commutator

$$[\widehat{\Lambda}, \widehat{\Lambda}'] = \frac{1}{2}\{\Lambda_a(x) \stackrel{\star}{,} \Lambda'_b(x)\}[T^a, T^b] + \frac{1}{2}[\Lambda_a(x) \stackrel{\star}{,} \Lambda'_b(x)]\{T^a, T^b\} \tag{5}$$

of two Lie algebra-valued noncommutative gauge parameters $\widehat{\Lambda} = \Lambda_a(x)T^a$ and $\widehat{\Lambda}' = \Lambda'_a(x)T^a$ does not close in Lie. For noncommutative SU(N) the Lie algebra traceless condition is incompatible with commutator. So, for noncommutative gauge transformation we have extension to the enveloping algebra-valued gauge transformation.

Seiberg-Witten map:
Closing condition for gauge transformation algebra are homogenus differential equations, which are solved by iteration, order by order in noncommutative parameter θ. Solutions are known as Seiberg-Witten map. Hermicity condition for the fields, up to the first order in Seiberg-Witten expansion, gives for gauge parameter, gauge and fermion fields the following expressions:

$$\begin{aligned}
\widehat{\Lambda} &= \Lambda + \frac{h}{4}\theta^{\mu\nu}\{V_\nu, \partial_\mu\Lambda\} + ... \\
\widehat{V}_\mu &= V_\mu + \frac{h}{4}\theta^{\alpha\beta}\{\partial_\alpha V_\mu + F_{\alpha\mu}, V_\beta\} + ... \\
\widehat{\psi} &= \psi - \frac{h}{2}\,\theta^{\alpha\beta}\left(V_\alpha\,\partial_\beta - \frac{\mathrm{i}}{4}\,[V_\alpha, V_\beta]\right)\psi + ...
\end{aligned} \tag{6}$$

Extended noncommutative gauge field theory
Commutative GFT, that are renormalizable with minimal coupling, are extended in the same minimal fashion to the NC space with deformed gauge transformations. These deformations are not unique. For instance deformed action S_g depends on the choice of representation. This derives from the fact that $\widehat{F}^{\mu\nu}$ is enveloping algebra not Lee algebra valued. So called extended gauge-invariant action is:

$$\begin{aligned} S_{\rm NC} &= S_g + S_\psi = -\frac{1}{2}\,{\rm Tr}\int d^4x\,\left(1-\frac{a-1}{2}h\theta_{\rho\sigma}\star\widehat{F}^{\rho\sigma}\right)\star\widehat{F}_{\mu\nu}\star\widehat{F}^{\mu\nu} \\ &\quad + {\rm i}\int {\rm d}^4x\,\widehat{\bar{\varphi}}\star\bar{\sigma}^\mu(\partial_\mu+{\rm i}\widehat{A}_\mu)\star\widehat{\varphi}\,. \end{aligned} \tag{7}$$

The trace Tr in S_g is over all representations. $\widehat{\varphi}$'s are the noncommutative Weyl spinors. Applying Seiberg-Witten map on the above action up to first order in θ we obtain 'minimal' actions

$$\begin{aligned} S_g &= -\frac{1}{2}{\rm Tr}\int d^4x\;F_{\mu\nu}F^{\mu\nu} \\ &\quad + h\,\theta^{\rho\sigma}{\rm Tr}\int d^4x\;\left[\left(\frac{a}{4}F_{\rho\sigma}F_{\mu\nu}-F_{\rho\mu}F_{\sigma\nu}\right)F^{\mu\nu}\right], \\ S_\psi &= {\rm i}\int d^4x\;\bar{\varphi}\sigma^\mu(\partial_\mu+{\rm i}A_\mu)\varphi \\ &\quad - \frac{h}{8}\theta^{\mu\nu}\Delta^{\alpha\beta\gamma}_{\mu\nu\rho}\int d^4x\;F_{\alpha\beta}\;\bar{\varphi}\;\bar{\sigma}^\rho(\partial_\gamma+{\rm i}A_\gamma)\varphi\,, \\ \Delta^{\alpha\beta\gamma}_{\mu\nu\rho} &= \varepsilon^{\alpha\beta\gamma\lambda}\varepsilon_{\lambda\mu\nu\rho}\,. \end{aligned} \tag{8}$$

Clearly we do not know the meaning of 'minimal coupling concept' for some NCGFT in the NC space. However, renormalization is the principle that help us to find such acceptable couplings. We learned that the renormalizability condition of some specific NCGFT requires introduction of the higher order noncommutative gauge interaction by expanding general NC action in terms of NC field strengths. This of course extends 'NC minimal coupling' of the gauge action S_g in (7) to higher order; with a being free parameter determining renormalizable deformation. This was possible due to the symmetry property of an object $\theta_{\rho\sigma}\star\widehat{F}^{\rho\sigma}$. SW map for NC field strength up to the first order in $h\theta^{\mu\nu}$ than gives:

In the chiral fermion sector the choice of Majorana spinors for the U(1) case gives

$$\begin{aligned} S_\psi &= \frac{\rm i}{2}\int d^4x\Big[\,\bar{\psi}\gamma^\mu(\partial_\mu-{\rm i}\gamma_5A_\mu)\psi \\ &\quad + {\rm i}\frac{h}{8}\,\theta^{\mu\nu}\Delta^{\alpha\beta\gamma}_{\mu\nu\rho}F_{\alpha\beta}\,\bar{\psi}\;\gamma^\rho(\partial_\gamma-{\rm i}\gamma_5A_\gamma)\psi\,\Big]\,. \end{aligned} \tag{9}$$

For the SU(2) case relevant expressions are given in [17].

Proposed framework gives starting action for the gauge and fermion sectors. Requirement of renormalizability fixes the freedom parameter a. That is, the principle of renormalization determines NC renormalizable deformation. Trace of three generators in the above action lead to dependence of the gauge group representation and the choice of the trace corresponds to the choice of the group representation.

The gauge sector of minimal NCSM
Here we choose vector field in the adjoint representation, i.e. using a sum of three traces over the standard model gauge group we obtain the following action in terms of physical fields,

$$\begin{aligned} S_g^{\mathrm{mNCSM}} &= -\frac{1}{2}\int d^4x \Big[\Big(\frac{1}{2}\mathcal{A}_{\mu\nu}\mathcal{A}^{\mu\nu} + \mathrm{Tr}\,\mathcal{B}_{\mu\nu}\mathcal{B}^{\mu\nu} + \mathrm{Tr}\,G_{\mu\nu}G^{\mu\nu} \Big) \\ &\quad -\frac{1}{2}\, g_s\, d^{abc}\, h\theta^{\rho\sigma} \left(\frac{a}{4} G^a_{\rho\sigma} G^b_{\mu\nu} - G^a_{\rho\mu} G^b_{\sigma\nu} \right) G^{\mu\nu,c} \Big] , \end{aligned} \tag{10}$$

where d^{abc} are totally symmetric SU(3) group coefficients which come from the trace in (8). The $\mathcal{A}_{\mu\nu}$, $\mathcal{B}_{\mu\nu}(= B^a_{\mu\nu} T^a_L)$ and $G_{\mu\nu}(= G^a_{\mu\nu} T^a_S)$ denote the U(1), $\mathrm{SU(2)_L}$ and $\mathrm{SU(3)_c}$ field strengths, respectively.

For adjoint representation their is no new neutral electroweak triple gauge boson interactions.

The nmNCSM gauge sector action
is obtained by taking trace over all massive particle multiplets with different quantum numbers in the model that have covariant derivative acting on them; five multiplets for each generation of fermions and one Higgs multiplet. Here V_μ is the standard model gauge potential:

$$V^\mu = g'\mathcal{A}^\mu(x) Y + g \sum_{a=1}^{3} B^\mu_a(x) T^a_L + g_s \sum_{b=1}^{8} G^\mu_b(x) T^b_S \,. \tag{11}$$

For detailes of the model see [6].

The interactions Lagrangian's in terms of physical fields and effective couplings are [6]:

$$\mathcal{L}^\theta_{\gamma\gamma\gamma} = \frac{e}{4}\sin 2\theta_W\, \mathrm{K}_{\gamma\gamma\gamma} h\theta^{\rho\tau} A^{\mu\nu}\left(aA_{\mu\nu}A_{\rho\tau} - 4A_{\mu\rho}A_{\nu\tau}\right) , \tag{12}$$

$$\begin{aligned} \mathcal{L}^\theta_{Z\gamma\gamma} &= \frac{e}{4}\sin 2\theta_W\, \mathrm{K}_{Z\gamma\gamma}\, h\theta^{\rho\tau}\, [2Z^{\mu\nu}\left(2A_{\mu\rho}A_{\nu\tau} - aA_{\mu\nu}A_{\rho\tau}\right) \\ &\quad + 8Z_{\mu\rho}A^{\mu\nu}A_{\nu\tau} - aZ_{\rho\tau}A_{\mu\nu}A^{\mu\nu}] \,, \quad \text{ect.}, \end{aligned} \tag{13}$$

$$\mathrm{K}_{\gamma\gamma\gamma} = \frac{1}{2}\, gg'(\kappa_1 + 3\kappa_2) \,, \qquad \mathrm{K}_{Z\gamma\gamma} = \frac{1}{2}\, \left[g'^2\kappa_1 + \left(g'^2 - 2g^2\right)\kappa_2 \right] , \quad \text{ect.}$$

Renormalization

One-loop renormalization is performed by using the background field method (BFM). Advantage of the BFM is the guarantee of covariance, because by doing the path integral the local symmetry of the quantum field Φ_V is fixed, while the gauge symmetry of the background field ϕ_V is manifestly preserved. Quantization is performed by the functional integration over the quantum vector field $\mathbf{\Phi}_V$ in the saddle-point approximation around classical (background) configuration. For case $\phi_V = constant$, the main contribution to the functional integral is given by the Gaussian integral. Split the vector potential into the classical background plus the quantum-fluctuation parts, that is: We replace, $\phi_V \to \phi_V + \mathbf{\Phi}_V$, and than compute the terms quadratic in the quantum fields. Interactions are of the polynomial type.

Proper quantization requires the presence of the gauge fixing term $S_{\rm gf}[\phi]$. Adding to the SM part in the usual way, Feynman-Faddeev-Popov ghost appears in the effective action. Result of functional integration

$$\Gamma[\phi] = S_{\rm cl}[\phi] + S_{\rm gf}[\phi] + \Gamma^{(1)}[\phi]\,, \quad S_{\rm gf}[\phi] = -\frac{1}{2}\int {\rm d}^4x (D_\mu \mathbf{\Phi}^\mu_{\mathbf{V}})^{\mathbf{2}}, \tag{14}$$

produce the standard result of the commutative part of our action. The one-loop effective part $\Gamma^{(1)}[\phi]$ is given by

$$\Gamma^{(1)}[\phi] = \frac{\rm i}{2}\log\det S^{(2)}[\phi] = \frac{\rm i}{2}{\rm Tr}\log S^{(2)}[\phi]\,, \tag{15}$$

where $S^{(2)}[\phi]$ is the second functional derivative of a classical action.

The one-loop effective action computed by using background field method gives noncommutative vertices; see detailes in [13, 14, 15, 17].

Renormalization of nmNCSM

Divergences for $\mathrm{U}(1)_{\mathrm{Y}}-\mathrm{SU}(2)_{\mathrm{C}}$ and $\mathrm{U}(1)_{\mathrm{Y}}-\mathrm{SU}(3)_{\mathrm{C}}$ mixed noncommutative terms are

$$\begin{aligned}
\Gamma^{(1)}_{\rm div} &= \frac{11}{3(4\pi)^2\epsilon}\int d^4x B^i_{\mu\nu}B^{\mu\nu i} + \frac{11}{2(4\pi)^2\epsilon}\int d^4x G^a_{\mu\nu}G^{\mu\nu a} \\
&+ \frac{4}{3(4\pi)^2\epsilon} g' g^2 \kappa_2 (3-a) h\theta^{\mu\nu}\int d^4x (\frac{1}{4}f_{\mu\nu}B^i_{\rho\sigma}B^{\rho\sigma i} - f_{\mu\rho}B^i_{\nu\sigma}B^{\rho\sigma i}) \\
&+ \frac{6}{3(4\pi)^2\epsilon} g' g_S^2 \kappa_3 (3-a) h\theta^{\mu\nu}\int d^4x (\frac{1}{4}f_{\mu\nu}G^a_{\rho\sigma}G^{\rho\sigma a} - f_{\mu\rho}G^a_{\nu\sigma}G^{\rho\sigma a})\,.
\end{aligned} \tag{16}$$

Renormalization is obtained via counter-terms and for the obvious choice $a = 3$, giving bare Lagrangian. Constants κ_1, κ_2 and κ_3 remain unchanged under renormalization if specific replacement in $1/g_i^2$ couplings were applied, see [14]. Since, for $a = 3$, our Lagrangian is free from divergences at one-loop noncommutative deformation parameter h need not be renormalized.

Renormalization of NC SU(N) gauge theory

Choosing vector field in the adjoint representation SU(N) we obtain the starting Lagrangian (10), where now d^{abc} are totally symmetric SU(N) group coefficients.

Renormalization of the theory is obtained by canceling divergences. To have that the counter terms should be added to the starting action, which than produces the bare Lagrangian which has two solutions: $a = 1$ and $a = 3$ [15].

The case $a = 1$ corresponds to previous result [13] and the deformation parameter h need not to be renormalized. Renormalizability is, in this case, obtained through the known renormalization of gauge fields and coupling constant only.

However the case $a = 3$ is different since additional divergences can be absorbed only into the noncommutative deformation parameter h. That is that h has to be renormalized. The bare gauge field, the coupling constant and the noncommutative deformation parameter are [15]:

$$V_0^\mu = V^\mu \sqrt{1 + \frac{22Ng^2}{3(4\pi)^2\epsilon}}, \quad g_0 = \frac{g\mu^{\epsilon/2}}{\sqrt{1 + \frac{22Ng^2}{3(4\pi)^2\epsilon}}}, \quad h_0 = \frac{h}{1 - \frac{2Ng^2}{3(4\pi)^2\epsilon}}\,. \qquad (17)$$

The necessity of the h renormalization jeopardizes previous hope that the NC SU(N) gauge theory might be renormalizable to all orders in $\theta^{\mu\nu}$. Above results are also valid for the minimal NCSM gauge sector (10) with $N = 3$.

Ultraviolet asymptotic behavior of NC SU(N) gauge theory

Gauge coupling constant g and the NC deformation parameter h in our theory depend on energy i.e., the renormalization point μ. Both beta functions (β_g, β_h) are *negative* that is they decrease with increasing energy μ [15]. Solution to β_h shows that by increase of energy μ the NC deformation parameter h decreases [15]. From this follows necessity of the modification of Heisenberg uncertainty relations at high energy. String theory inspired modification

$$[x,p] = i\hbar(1 + \beta p^2) \quad \Rightarrow \quad \Delta x = \frac{\hbar}{2}(\frac{1}{\Delta p} + \beta\Delta p). \qquad (18)$$

show that for large momenta Δp (energy) distance Δx grows linearly. So large energies do not necessarily correspond to small distances, and running h does not imply that noncommutativity vanishes at small distances. This is related to UV/IR correspondence. From Eq. (17) and $h = 1/\Lambda_{\rm NC}$ we have

$$h(\mu) = \frac{1}{\Lambda^2_{\rm NC}(\mu)} \quad \Rightarrow \quad \Lambda_{\rm NC}(\mu) = \Lambda_{\rm NC}\sqrt{\ln\frac{\mu}{\Lambda}}\,. \qquad (19)$$

This way, via RGE, the scale of noncommutativity $\Lambda_{\rm NC}$ becomes the running scale of noncommutativity [15]. However it receives very small change when energy μ increases.

The 4ψ divergences for NC chiral fermions in U(1) and SU(2) cases

The one-loop effective action is computed from Eq. (9) by using background field method.

Our computations shows that divergent contributions from relevant terms are finite due to the structure of the momentum integrals in both, the U(1) and the SU(2), cases. For NC chiral electrodynamics, with Majorana spinors and with the usual definition for the supertrace STr, [25], we have used chiral fermions in the fundamental representation of SU(2). Choosing Majorana spinors we apparently break the SU(2) symmetry, and consequently we have to work in the framework of the components for the vector potential. Now of course, Majorana $\binom{\psi_1}{\psi_2}$ is not a SU(2) doublet.

Clearly, we conclude that direct computations by using BFM confirms results of the symmetry analysis for the 4ψ divergent terms, which, due to their U(1) invariance, have to be zero [17]. The same symmetry arguments holds also SU(2) terms, i.e. they both vanish identically too.

Limits on the noncommutativity scale

From the gauge-invariant amplitude for $Z \to \gamma\gamma$, gg decays in momentum space and for Z boson at rest and for $a = 3$, we have found the following branching ratios [18],

$$BR_{(Z\to\gamma\gamma)} = \tau_Z \frac{\alpha}{4} \frac{M_Z^5}{\Lambda_{\rm NC}^4} \sin^2 2\theta_W {\rm K}^2_{Z\gamma\gamma} (\vec{E}_\theta^2 + \vec{B}_\theta^2) = \frac{1}{8} \frac{{\rm K}^2_{Z\gamma\gamma}}{{\rm K}^2_{Zgg}} BR_{(Z\to gg)}, \quad (20)$$

where τ_Z is the Z boson lifetime. LHC experimental possibilities for $Z \to \gamma\gamma$ we analyze by using the CMS Physics Technical Design Report [21, 22]. We have found that for 10^7 events of $Z \to e^+e^-$ for 10 fb^{-1} in 2 years of LHC running and by assuming $BR(Z \to \gamma\gamma) \sim 10^{-8}$ and using $BR(Z \to e^+e^-) = 0.03$ about ~ 3 events of $Z \to \gamma\gamma$ decays should be found. However, note that background sources (CMS Note 2006/112, Fig.3) could potentially be a big problem. For example study for $Higgs \to \gamma\gamma$ shows that, when e^- from $Z \to e^+e^-$ radiates very high energy Bremsstrahlung photon into pixel detector, for similar energies of e^- and γ, there is a huge probability of misidentification of e^- with γ. Second, the irreducible di-photon background may also kill the signal. The $Z \to gg$ decay was discussed in [6].

Finally, note that after 10 years of LHC running integrated luminosity would reach $\sim$ 1000 fb^{-1}. In that case and from bona fide reasonable assumption $BR(Z \to \gamma\gamma) \sim 10^{-8}$ one would find 300 events of $Z \to \gamma\gamma$ decays, or one would have ~ 3 events with $BR(Z \to \gamma\gamma) \sim 10^{-10}$. From above it follows that, in the later case, the lower bound on the scale of noncommutativity would be $\Lambda_{\rm NC} > 1$ TeV.

Limits on the scale of noncommutativity in high energy particle physics are coming from the analysis of decay and scattering experiments.

Considering SM forbidden decays, recently we have found the following lower limit $\Lambda_{\rm NC} > 1$ TeV [18] from $Z \to \gamma\gamma$ decay. Note here that earlier limits obtained from $\gamma_{\rm pl} \to \nu\bar{\nu}$ decay (astrophysics analysis) produces $\Lambda_{\rm NC} > 81$ GeV [19] while from the SM forbidden $J/\psi \to \gamma\gamma$ and $K \to \pi\gamma$ [20] decays we obtain $\Lambda_{\rm NC} > 9$ GeV, and $\Lambda_{\rm NC} > 43$ GeV, respectively. Last two bounds are not usefull due to the too high lower limit of the relevant branching ratios.

Scattering experiments [23] support the above obtained limits. From annihilation $\gamma\gamma \to \bar{f}f$ it was found $\Lambda_{\rm NC} > 200$ GeV, which is a bit to low. However, from $\bar{f}f \to Z\gamma$ unelastic scattering experiments there is very interesting limit $\Lambda_{\rm NC} > 1$ TeV.

Summary and Conclusion

Principle of renormalizability implemented on our θ-expanded NCGFT led us to well defined deformation via introduction of higher order noncommutative action class for the gauge sectors of the mNCSM, nmNCSM and NC SU(N) models. This extension was parametrized by generically free parameter a:

$$S_g = -\frac{1}{2}\mathrm{Tr}\int d^4x \left(1 + \mathrm{i}(a-1)\,\widehat{x}_\rho \star \widehat{x}_\sigma \star \widehat{F}^{\rho\sigma}\right) \star \widehat{F}_{\mu\nu} \star \widehat{F}^{\mu\nu}\,. \tag{21}$$

We have found the following properties of the above models with respect to renormalization procedure:

- Renormalization principle is fixing the freedom parameter a for our θ-expanded NC GFT.
- Divergences cancel differently than in commutative GFT and this depends on the representations.
- Gauge sector of the nmNCSM, which produces SM forbidden $Z \to \gamma\gamma$ decay, is renormalizable and *finite* for $a = 3$. Due to this finiteness no renormalization of h necessary.
- Noncommutative SU(N) gauge theory is renormalizable for $a = 1$ and $a = 3$. The case $a = 1$ corresponds to the earlier obtained result [13]. However, in the case $a = 3$ additional divergences appears and had to be absorbed through the renormalization of the noncommutative deformation h. Hence, in the case of noncommutative SU(N) the noncommutativity deformation parameter h had to be renormalized and it is *asymptotically free*, opposite to the previous expectations. The same is valid for mNCSM.
- The solution $a = 3$, while shifting the model to the higher order, i.e. while extending 'NC minimal coupling', hints into the discovery of the key role of the higher noncommutative gauge interaction in one-loop renormalizability of classes of NCGFT at the first order in θ.

- Our computations also confirms symmetry arguments that for noncommutative chiral electrodynamics, that is the U(1) case with Majorana spinors, the 4ψ divergent part vanishes. For noncommutative chiral fermions in the fundamental representation of SU(2) with Majorana spinors the 4ψ divergent part vanishes due to the SU(2) invariance. So, for noncommutative U(1) and SU(2) chiral fermion models typical 4ψ-divergence is *absent*, contrary to the earlier results obtained for Dirac fermions [24, 25].
- There is similarity to noncommutative ϕ^4 theory [26].
- Note that the renormalizability principle could help to minimize or even cancel most of the ambiguities of the higher order Seiberg-Witten maps [27].
- Finally, phenomenological results, as the standard model forbidden $Z \to \gamma\gamma$ decay, are *robust* due to the one-loop renormalizability and *finiteness* of the nmNCSM gauge sector [14, 18].

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