Improved relations between type I and type IIB background fields*

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Abstract

In this paper we consider pure spinor formulation of type IIB superstring theory in the presence of constant background fields: graviton $G_{\mu\nu}$, antisymmetric Neveu-Schwarz (NS-NS) field $B_{\mu\nu}$, two same chirality gravitinos, ψ^α_μ and $\bar\psi^\alpha_\mu$, and Ramond-Ramond (R-R) field strength $F^{\alpha\beta}$. We choose the boundary conditions which preserve $N=1$ SUSY from the initial $N=2$. Effective theory, as initial one on the solution of boundary conditions, is even under world-sheet parity transformation Ω. We recognize effective theory as type I closed superstring theory which propagates in effective background depending linearly on Ω even fields: graviton, mean value of two gravitinos, and antisymmetric part of R-R field strength, and bilinearly on Ω odd fields: NS-NS field $B_{\mu\nu}$, difference between two gravitinos, and the symmetric part of R-R field strength. These quadratic terms in background fields represent our improvements. We also considered the contribution of background fields to the non(anti)commutativity of superspace coordinates, bosonic ones x^μ and same chirality fermionic coordinates, θ^α and $\bar\theta^\alpha$.

1. Introduction

It is known that type I superstring theory can be obtained from type IIB superstring theory as a projection on the states that are even under world-sheet parity transformation $\Omega : \sigma \to -\sigma$. After this orientifold projection graviton $G_{\mu\nu}$ and dilaton Φ from NS-NS sector, the sum of two same chirality gravitions $\psi^\alpha_{+\mu} = \psi^\alpha_\mu + \bar\psi^\alpha_\mu$ and sum of two dilatinos $\lambda^\alpha_+ = \lambda^\alpha + \bar\lambda^\alpha$

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from NS-R sector and two rank antisymmetric tensor $C_{\mu\nu}$ from R-R sector survive. The states that are odd under Ω transformations: antisymmetric tensor $B_{\mu\nu}$ from NS-NS sector, difference of two gravitinos $\psi^\alpha_{-\mu} = \psi^\alpha_\mu - \bar{\psi}^\alpha_\mu$ and dilatinios $\lambda^\alpha_- = \lambda^\alpha - \bar{\lambda}^\alpha$ from NS-R sector, and scalar C_0 and four rank antisymmetric tensor $C_{\mu\nu\rho\sigma}$ with self dual field strength from R-R sector, are eliminated by above projection.

We are going to investigate pure spinor formulation of type IIB superstring [1], in the form considered in Ref.[2] which corresponds to constant background fields: graviton $G_{\mu\nu}$, antisymmetric field $B_{\mu\nu}$, two gravitinos ψ^α_μ and $\bar{\psi}^\alpha_\mu$, and R-R field strength $F^{\alpha\beta}$. In this approach dilaton Φ and two dilatinos λ^α and $\bar{\lambda}^\alpha$ are set to zero.

We choose Neumann boundary conditions for bosonic coordinates x^μ and $(\theta^\alpha - \bar{\theta}^\alpha)|_0^\pi = 0$ for fermionic coordinates in order to preserve $N = 1$ SUSY from initial $N = 2$ SUSY in type IIB theory. It occurs that effective theory, initial theory on the solution of boundary conditions, is even under world-sheet parity transformation Ω, and we can identify it as type I closed superstring theory. We show that fields $B_{\mu\nu}$ and $\psi^\alpha_{-\mu}$, which describe states odd under parity transformations, are not completely eliminated, but they survive as quadratic terms in type I superstring background [3].

We also discuss noncommutativity properties of the open IIB superstring theory with above choice of boundary conditions. In bosonic case with constant background fields, Dp-brane, on which the open string endpoints are attached, is noncommutative manifold [4, 5, 6], and the noncommutativity parameter is proportional to the antisymmetric NS-NS field $B_{\mu\nu}$. In supersymmetric case the presence of nontrivial background fields odd under Ω transformations: antisymmetric field $B_{\mu\nu}$, difference of two gravitionos $\psi^\alpha_{-\mu} = \psi^\alpha_\mu - \bar{\psi}^\alpha_\mu$, scalar $C_{(0)}$ and four rank antisymmetric tensor $C_{(4)}$ leads to noncommutativity of superspace coordinates. This is consequence of the fact that the solution of original supercoordinates on the boundary conditions depends not only on the effective ones, but also on their momenta.

At the end we give some concluding remarks.

2. Pure spinor formulation of IIB superstring theory

The pure spinor formulation of type IIB superstring theory of Ref.[2] is described by the action

$$
\begin{aligned}
S = \kappa \int_\Sigma d^2\xi \left[\frac{1}{2}\eta^{ab} G_{\mu\nu} + \varepsilon^{ab} B_{\mu\nu}\right] \partial_a x^\mu \partial_b x^\nu \\
+ \int_\Sigma d^2\xi \left[-\pi_\alpha(\partial_\tau - \partial_\sigma)(\theta^\alpha + \psi^\alpha_\mu x^\mu) + (\partial_\tau + \partial_\sigma)(\bar{\theta}^\alpha + \bar{\psi}^\alpha_\mu x^\mu)\bar{\pi}_\alpha\right] \\
+ \int_\Sigma d^2\xi \frac{1}{2\kappa} \pi_\alpha F^{\alpha\beta} \bar{\pi}_\beta \, ,
\end{aligned}
\tag{1}
$$

where the world sheet Σ is parametrized by $\xi^a = (\xi^0 = \tau\,, \xi^1 = \sigma)$, and $D = 10$ dimensional space-time is parameterized by coordinates x^μ ($\mu = 0, 1, 2, \ldots, 9$). The fermionic part of superspace is spanned by same chirality fermionic coordinates θ^α and $\bar\theta^\alpha$, while the variables π_α and $\bar\pi_\alpha$ are their canonically conjugated momenta. The fermionic coordinates and momenta are 16 component Majorana-Weyl spinors.

The connection between two descriptions of R-R sector, with field strength $F^{\alpha\beta}$ and with antisymmetric tensors $C_{(0)}$, $C_{(2)}$ and $C_{(4)}$ [7], can be established, where $C_{(k)}$ denotes k-rank antisymmetric tensor. It is known that bispinor $F^{\alpha\beta}$ can be expand into complete set of antisymmetric gamma matrices

$$F^{\alpha\beta} = \sum_{k=0}^{D} \frac{1}{k!} F_{(k)} \Gamma^{\alpha\beta}_{(k)}\,, \quad \left[\Gamma^{\alpha\beta}_{(k)} = (C\Gamma^{[\mu_1 \ldots \mu_k]})^{\alpha\beta}\right] \tag{2}$$

where C is charge conjugation operator. The bispinor $F^{\alpha\beta}$ also satisfy chirality condition, $\Gamma_{11} F = -F\Gamma_{11}$, and, consequently, type IIB theory contains only odd rank tensors $F_{(k)}$. Because of duality relation, the independent tensors are $F_{(1)}$, $F_{(3)}$ and self-dual part of $F_{(5)}$. Using mass-shell condition these tensors can be solved in terms of potentials $F_{(k)} = dC_{(k-1)}$, so that IIB theory contains just the potentials $C_{(0)}$, $C_{(2)}$ and $C_{(4)}$. Symmetric part of $F^{\alpha\beta}$, $F_s^{\alpha\beta} = \frac{1}{2}(F^{\alpha\beta} + F^{\beta\alpha})$, corresponds to the potentials C_0 and $C_{\mu\nu\rho\sigma}$, and antisymmetric part, $F_a^{\alpha\beta} = \frac{1}{2}(F^{\alpha\beta} - F^{\beta\alpha})$, corresponds to the potential $C_{\mu\nu}$.

Momentum canonically conjugated to x^μ has the form

$$\pi_\mu \equiv \frac{\partial \mathcal{L}}{\partial \dot x^\mu} = \kappa (G_{\mu\nu} \dot x^\nu - 2B_{\mu\nu} x'^\nu) - \pi_\alpha \psi^\alpha_\mu + \bar\psi^\alpha_\mu \bar\pi_\alpha\,. \tag{3}$$

Using the definition of canonical Hamiltonian, $\mathcal{H}_c = \dot x^\mu \pi_\mu + \dot\theta^\alpha \pi_\alpha + \dot{\bar\theta}^\alpha \bar\pi_\alpha - \mathcal{L}$, we have

$$H_c = \int d\sigma \mathcal{H}_c\,, \quad \mathcal{H}_c = T_- - T_+\,, \quad T_\pm = t_\pm - \tau_\pm\,, \tag{4}$$

where

$$\begin{aligned} t_\pm &= \mp \frac{1}{4\kappa} G^{\mu\nu} I_{\pm\mu} I_{\pm\nu}\,, \quad I_{\pm\mu} = \pi_\mu + 2\kappa \Pi_{\pm\mu\nu} x'^\nu + \pi_\alpha \psi^\alpha_\mu - \bar\psi^\alpha_\mu \bar\pi_\alpha\,, \\ \tau_+ &= (\theta'^\alpha + \psi^\alpha_\mu x'^\mu)\pi_\alpha - \frac{1}{4\kappa} \pi_\alpha F^{\alpha\beta} \bar\pi_\beta\,, \\ \tau_- &= (\bar\theta'^\alpha + \bar\psi^\alpha_\mu x'^\mu)\bar\pi_\alpha + \frac{1}{4\kappa} \pi_\alpha F^{\alpha\beta} \bar\pi_\beta\,. \end{aligned} \tag{5}$$

From the algebra of currents

$$\begin{aligned} &\{I_{\pm\mu}(\sigma)\,, I_{\pm\nu}(\bar\sigma)\} = \pm 2\kappa G_{\mu\nu} \delta'\,, \qquad \{I_{\pm\mu}(\sigma)\,, I_{\mp\nu}(\bar\sigma)\} = 0\,, \\ &\left[\delta' \equiv \partial_\sigma \delta(\sigma - \bar\sigma)\right] \end{aligned} \tag{6}$$

it follows that components $T_\pm$ satisfy Virasoro algebra

$$\{T_\pm(\sigma)\,,T_\pm(\bar\sigma)\} = -\left[T_\pm(\sigma)+T_\pm(\bar\sigma)\right]\delta'\,,\quad \{T_\pm(\sigma)\,,T_\mp(\bar\sigma)\}=0\,. \tag{7}$$

The Poisson bracket between canonical Hamiltonian and current $I_{\pm\mu}$ is proportional to its sigma derivative

$$\{H_c\,,I_{\pm\mu}\} = \mp I'_{\pm\mu}\,, \tag{8}$$

which means that current $I_{\pm\mu}$ depends on the corresponding chiral coordinate $\sigma_\pm = \tau\pm\sigma$.

3. Supersymmetric boundary conditions as canonical constraints

In this section we derive boundary conditions at string endpoints in canonical form. We apply Dirac consistency procedure and obtain infinite set of constraints. We show that all of them are of the second class.

3.1. Canonical derivation of boundary conditions

Varying Hamiltonian H_c we obtain

$$\delta H_c = \delta H_c^{(R)} - [\gamma_\mu^{(0)}\delta x^\mu + \pi_\alpha\delta\theta^\alpha + \delta\bar\theta^\alpha\bar\pi_\alpha]\big|_0^\pi\,, \tag{9}$$

where $\delta H_c^{(R)}$ is regular term of the form

$$\delta H_c^{(R)} = \int d\sigma(A_\mu\delta x^\mu + B^\mu\delta\pi_\mu + C_\alpha\delta\theta^\alpha + D_\alpha\delta\bar\theta^\alpha + E^\alpha\delta\pi_\alpha + F^\alpha\delta\bar\pi_\alpha)\,, \tag{10}$$

and

$$\gamma_\mu^{(0)} = \Pi_{+\mu}{}^\nu I_{-\nu} + \Pi_{-\mu}{}^\nu I_{+\nu} + \pi_\alpha\psi_\mu^\alpha + \bar\psi_\mu^\alpha\bar\pi_\alpha\,. \tag{11}$$

From the condition that canonical Hamiltonian as time translation generator must have well defined derivatives in its variables [8], we have

$$\left[\gamma_\mu^{(0)}\delta x^\mu + \pi_\alpha\delta\theta^\alpha + \delta\bar\theta^\alpha\bar\pi_\alpha\right]\Big|_0^\pi = 0\,. \tag{12}$$

For bosonic coordinates x^μ we choose Neumann boundary conditions, allowing arbitrary variations δx^μ at the string endpoints, which gives

$$\gamma_\mu^{(0)}\big|_0^\pi = 0\,. \tag{13}$$

In order to preserve $N=1$ SUSY of the initial $N=2$ SUSY, following [2, 9], for fermionic coordinates we take

$$(\theta^\alpha - \bar\theta^\alpha)\Big|_0^\pi = 0\,, \tag{14}$$

which produces

$$(\pi_\alpha - \bar\pi_\alpha)\big|_0^\pi = 0\,. \tag{15}$$

3.2. Consistency of constraints

We will treat the boundary conditions (13)-(15) as canonical constraints. In order to write boundary conditions in suitable form, we introduce variables symmetric and antisymmetric under Ω-parity transformation ($\Omega : \sigma \to -\sigma$). Defining projectors

$$P_s = \frac{1}{2}(1+\Omega)\,, \qquad P_a = \frac{1}{2}(1-\Omega)\,, \tag{16}$$

for bosonic variables we will use standard notation [10, 11]

$$q^\mu(\sigma) \equiv P_s x^\mu(\sigma)\,, \qquad \tilde{q}^\mu(\sigma) \equiv P_a x^\mu(\sigma)\,, \tag{17}$$

$$p_\mu(\sigma) \equiv P_s \pi_\mu(\sigma)\,, \qquad \tilde{p}_\mu(\sigma) \equiv P_a \pi_\mu(\sigma)\,, \tag{18}$$

while for fermionic ones we will explicitly apply P_s and P_a.

The consistency procedure for $\gamma_\mu^{(0)}$ gives infinite sets of constraints on each string endpoint, $\gamma_\mu^{(n)}(0)$ and $\gamma_\mu^{(n)}(\pi)$, where

$$\begin{aligned}\gamma_\mu^{(n)} \equiv \left\{H_c\,, \gamma_\mu^{(n-1)}\right\} &= \Pi_{+\mu}{}^\nu \partial_\sigma^n I_{-\nu} + (-1)^n \Pi_{-\mu}{}^\nu \partial_\sigma^n I_{+\nu} \\ &+ (-1)^n \partial_\sigma^n \pi_\alpha \psi_\mu^\alpha + \bar{\psi}_\mu^\alpha \partial_\sigma^n \bar{\pi}_\alpha\,.\end{aligned} \tag{19}$$

At $\sigma = 0$ they can be rewritten in the compact σ-dependent form

$$\begin{aligned}\Gamma_\mu(\sigma) \equiv \sum_{n=0}^{\infty} \frac{\sigma^n}{n!} \gamma_\mu^{(n)}(\sigma=0) &= \Pi_{+\mu}{}^\nu I_{-\nu}(\sigma) + \Pi_{-\mu}{}^\nu I_{+\nu}(-\sigma) \\ &+ \pi_\alpha(-\sigma)\psi_\mu^\alpha + \bar{\psi}_\mu^\alpha \bar{\pi}_\alpha(\sigma) = \tilde{p}_\mu + 2(BG^{-1})_\mu{}^\nu p_\nu - \kappa G^{eff}_{\mu\nu} \tilde{q}'^\nu \\ &+ 2\Pi_{+\mu}{}^\nu P_s \pi_\alpha(\sigma) \psi_\nu^\alpha - 2\Pi_{-\mu}{}^\nu \bar{\psi}_\nu^\alpha P_s \bar{\pi}_\alpha(\sigma)\,.\end{aligned} \tag{20}$$

To consider fermionic constraints it is useful to define the recurrent equation

$$A^{(n+1)} \equiv \{H_c\,, A^{(n)}\}\,, \quad (n \geq 0) \tag{21}$$

where $A(\sigma)$ is an arbitrary function of σ. Applying this to expressions $A^{(0)} = (\theta^\alpha\,, \bar{\theta}^\alpha\,, \pi_\alpha\,, \bar{\pi}_\alpha)$ we obtain, respectively

$$\Theta^\alpha_{(2n)} = \partial_\sigma^{(2n)} \theta^\alpha\,, \tag{22}$$

$$\begin{aligned}\Theta^\alpha_{(2n+1)} &= -\partial_\sigma^{(2n+1)}\theta^\alpha - \psi_\mu^\alpha \partial_\sigma^{(2n+1)} x^\mu - \frac{1}{2\kappa} F^{\alpha\beta} \partial_\sigma^{(2n)} \bar{\pi}_\beta \\ &+ \frac{1}{2\kappa} G^{\mu\nu} \partial_\sigma^{(2n)} (I_{+\mu} + I_{-\mu}) \psi_\nu^\alpha\,,\end{aligned} \tag{23}$$

$$\bar{\Theta}^\alpha_{(2n)} = \partial_\sigma^{(2n)} \bar{\theta}^\alpha\,, \tag{24}$$

$$\bar{\Theta}^{\alpha}_{(2n+1)} = \partial_\sigma^{(2n+1)}\bar{\theta}^\alpha + \bar{\psi}^\alpha_\mu \partial_\sigma^{(2n+1)} x^\mu + \frac{1}{2\kappa}\partial_\sigma^{(2n)}\pi_\beta F^{\beta\alpha}$$
$$+ \frac{1}{2\kappa} G^{\mu\nu}\partial_\sigma^{(2n)}(I_{+\mu} + I_{-\mu})\bar{\psi}^\alpha_\nu \,, \tag{25}$$

$$\Pi^{(n)}_\alpha = (-1)^n \partial_\sigma^{(n)}\pi_\alpha \,, \quad \bar{\Pi}^{(n)}_\alpha = \partial_\sigma^{(n)}\bar{\pi}_\alpha \,. \tag{26}$$

Defining the function

$$\mathcal{A}(\sigma) \equiv \sum_{n=0}^{\infty} \frac{\sigma^n}{n!} A^{(n)}(\sigma = 0) \,, \tag{27}$$

expressions (22)-(26) can be written in more compact form

$$\Theta^\alpha(\sigma) = \theta^\alpha(-\sigma) - \psi^\alpha_\mu \tilde{q}^\mu(\sigma) - \frac{1}{2\kappa} F^{\alpha\beta}\int_0^\sigma d\sigma_1 P_s \bar{\pi}_\beta$$
$$+ \frac{1}{2\kappa} G^{\mu\nu}\psi^\alpha_\mu \int_0^\sigma d\sigma_1 P_s (I_{+\nu} + I_{-\nu}) \,, \tag{28}$$

$$\bar{\Theta}^\alpha(\sigma) = \bar{\theta}^\alpha(\sigma) + \bar{\psi}^\alpha_\mu \tilde{q}^\mu(\sigma) + \frac{1}{2\kappa} F^{\beta\alpha}\int_0^\sigma d\sigma_1 P_s \pi_\beta$$
$$+ \frac{1}{2\kappa} G^{\mu\nu}\bar{\psi}^\alpha_\mu \int_0^\sigma d\sigma_1 P_s (I_{+\nu} + I_{-\nu}) \,, \tag{29}$$

$$\Pi_\alpha(\sigma) = \pi_\alpha(-\sigma) \,, \qquad \bar{\Pi}_\alpha(\sigma) = \bar{\pi}_\alpha(\sigma) \,. \tag{30}$$

From conditions $(\theta^\alpha - \bar{\theta}^\alpha)\big|_0 = 0$ and $(\pi_\alpha - \bar{\pi}_\alpha)\big|_0 = 0$, using the relations (28)-(30), we conclude that

$$\Gamma^\alpha(\sigma) = \Theta^\alpha(\sigma) - \bar{\Theta}^\alpha(\sigma) \,, \tag{31}$$

and

$$\Gamma^\pi_\alpha(\sigma) \equiv \Pi_\alpha(\sigma) - \bar{\Pi}_\alpha(\sigma) = \pi_\alpha(-\sigma) - \bar{\pi}_\alpha(\sigma) \,, \tag{32}$$

are also canonical constraints

The complete constraints at $\sigma = \pi$ can be obtained in similar way. They have the same form as constraints at $\sigma = 0$, where all variables depending on $-\sigma$ are replaced by the variables depending on $2\pi - \sigma$. That set of constraints is solved by 2π periodicity of all canonical variables as well as in Refs.[10, 11].

Let us denote all constraints with $\Gamma_A = (\Gamma_\mu \,, \Gamma^\alpha \,, \Gamma^\pi_\alpha)$. From

$$\{H_c \,, \Gamma_A\} = \Gamma'_A \approx 0 \,, \tag{33}$$

it follows that all constraints weakly commute with canonical Hamiltonian, so there are no more constraints in the theory and the consistency procedure is completed.

3.3. Classification of constraints

We will consider the quantity Γ'^{α} instead Γ^{α} as constraint in the theory, without effect on the final result. The full set of constraints we will denote with ${}^{\star}\Gamma_A = (\Gamma_\mu \,, \Gamma'^{\alpha} \,, \Gamma^{\pi}_{\alpha})$. We compute the algebra of the constraints

$$\{{}^{\star}\Gamma_A \,, {}^{\star}\Gamma_B\} = M_{AB}\delta' \,, \tag{34}$$

where the supermatrix M_{AB} is given by the expression

$$M_{AB} = \begin{pmatrix} (M_1)_{\mu\nu} & (M_2)_{\mu}{}^{\gamma}{}_{\delta} \\ (M_3)^{\alpha}{}_{\beta\nu} & (M_4)^{\alpha}{}_{\beta}{}^{\gamma}{}_{\delta} \end{pmatrix} = \left(\begin{array}{c|cc} (M_1)_{\mu\nu} & M_{\mu}{}^{\gamma} & \overline{M}_{\mu\delta} \\ \hline M^{\alpha}{}_{\nu} & M^{\alpha\gamma} & \overline{M}^{\alpha}{}_{\delta} \\ \overline{M}_{\beta\nu} & \overline{M}_{\beta}{}^{\gamma} & \tilde{M}_{\beta\delta} \end{array}\right) . \tag{35}$$

Here we introduced the notation

$$\begin{aligned} \{\Gamma_\mu(\sigma) \,, \Gamma_\nu(\bar{\sigma})\} &\equiv (M_1)_{\mu\nu}\delta' = -\kappa G^{eff}_{\mu\nu}\delta' \,, \\ \left\{\Gamma_\mu(\sigma) \,, \Gamma'^{\alpha}(\bar{\sigma})\right\} &\equiv M_{\mu}{}^{\alpha}\delta' = 2(\Psi_{eff})^{\alpha}_{\mu}\delta' \,, \end{aligned} \tag{36}$$

$$\begin{aligned} \{\Gamma_\mu(\sigma) \,, \Gamma^{\pi}_{\alpha}(\bar{\sigma})\} &\equiv \overline{M}_{\mu\alpha}\delta' = 0 \,, \\ \left\{\Gamma'^{\alpha}(\sigma) \,, \Gamma'^{\beta}(\bar{\sigma})\right\} &\equiv M^{\alpha\beta}\delta' = -\frac{1}{\kappa}F^{\alpha\beta}_{eff}\delta' \,, \end{aligned} \tag{37}$$

$$\begin{aligned} \left\{\Gamma'^{\alpha}(\sigma) \,, \Gamma^{\pi}_{\beta}(\bar{\sigma})\right\} &\equiv \overline{M}^{\alpha}{}_{\beta}\delta' = -2\delta^{\alpha}{}_{\beta}\delta' \,, \\ \left\{\Gamma^{\pi}_{\alpha}(\sigma) \,, \Gamma^{\pi}_{\beta}(\bar{\sigma})\right\} &\equiv \tilde{M}_{\alpha\beta}\delta' = 0 \,, \end{aligned} \tag{38}$$

where

$$\begin{aligned} G^{eff}_{\mu\nu} &= G_{\mu\nu} - 4B_{\mu\rho}G^{\rho\lambda}B_{\lambda\nu} \,, \quad (\Psi_{eff})^{\alpha}_{\mu} = \frac{1}{2}\psi^{\alpha}_{+\mu} + B_{\mu\rho}G^{\rho\nu}\psi^{\alpha}_{-\nu} \,, \\ F^{\alpha\beta}_{eff} &= F^{\alpha\beta}_{a} - \psi^{\alpha}_{-\mu}G^{\mu\nu}\psi^{\beta}_{-\nu} \,, \end{aligned} \tag{39}$$

and the fields $\psi^{\alpha}_{\pm\mu}$ are defined as

$$\psi^{\alpha}_{\pm\mu} = \psi^{\alpha}_{\mu} \pm \bar{\psi}^{\alpha}_{\mu} \,. \tag{40}$$

From the definition of superdeterminant

$$s\det M_{AB} = \frac{\det(M_1 - M_2M_4^{-1}M_3)}{\det M_4} \,, \tag{41}$$

and above expressions, we obtain

$$s\det M_{AB} \sim \det G^{eff}_{\mu\nu} \,. \tag{42}$$

Assuming that effective metric $G^{eff}_{\mu\nu}$ is nonsingular, all constraints originating from boundary conditions are of the second class.

4. Solution of constraints and noncommutativity

Solving second class constraints $\Gamma_\mu = 0$, $\Gamma^\alpha = 0$ and $\Gamma^\pi_\alpha = 0$, we obtain

$$x^\mu(\sigma) = q^\mu - 2\Theta^{\mu\nu}\int_0^\sigma d\sigma_1 p_\nu + 2\Theta^{\mu\alpha}\int_0^\sigma d\sigma_1 p_\alpha\,, \quad \pi_\mu = p_\mu\,, \tag{43}$$

$$\theta^\alpha(\sigma) = \frac{1}{2}(\xi^\alpha + \tilde{\xi}^\alpha) - \Theta^{\mu\alpha}\int_0^\sigma d\sigma_1 p_\mu - \Theta^{\alpha\beta}\int_0^\sigma d\sigma_1 p_\beta\,, \, \pi_\alpha = p_\alpha + \tilde{p}_\alpha \tag{44}$$

$$\bar{\theta}^\alpha(\sigma) = \frac{1}{2}(\xi^\alpha - \tilde{\xi}^\alpha) - \Theta^{\mu\alpha}\int_0^\sigma d\sigma_1 p_\mu - \Theta^{\alpha\beta}\int_0^\sigma d\sigma_1 p_\beta\,, \, \bar{\pi}_\alpha = p_\alpha - \tilde{p}_\alpha \tag{45}$$

where

$$\frac{1}{2}\,\xi^\alpha \equiv \theta^\alpha_s = \bar{\theta}^\alpha_s\,, \quad \tilde{\xi}^\alpha \equiv \theta^\alpha_a - \bar{\theta}^\alpha_a\,, \quad p_\alpha \equiv \pi^s_\alpha = \bar{\pi}^s_\alpha\,, \quad \tilde{p}_\alpha \equiv \pi^a_\alpha = -\bar{\pi}^a_\alpha \tag{46}$$

and

$$\Theta^{\mu\nu} = -\frac{1}{\kappa}(G^{-1}_{eff}BG^{-1})^{\mu\nu}\,, \quad \Theta^{\mu\alpha} = 2\Theta^{\mu\nu}(\Psi_{eff})^\alpha_\nu - \frac{1}{2\kappa}G^{\mu\nu}\psi^\alpha_{-\nu}\,, \tag{47}$$

$$\begin{aligned}\Theta^{\alpha\beta} = {} & \frac{1}{2\kappa}F^{\alpha\beta}_s + 4(\Psi_{eff})^\alpha_\mu\Theta^{\mu\nu}(\Psi_{eff})^\beta_\nu - \frac{1}{\kappa}\psi^\alpha_{-\mu}(G^{-1}BG^{-1})^{\mu\nu}\psi^\beta_{-\nu} \\ & + \frac{G^{\mu\nu}}{\kappa}\left[\psi^\alpha_{-\mu}(\Psi_{eff})^\beta_\nu + \psi^\beta_{-\mu}(\Psi_{eff})^\alpha_\nu\right]\,.\end{aligned} \tag{48}$$

Using the solutions of constraints (43)-(45) for center of mass variables (see Appendix C of Ref.[11]), we get the noncommutativity relations

$$\{x^\mu(\sigma)\,, x^\nu(\bar{\sigma})\} = \Theta^{\mu\nu}\Delta(\sigma+\bar{\sigma})\,, \tag{49}$$

$$\{x^\mu(\sigma)\,, \theta^\alpha(\bar{\sigma})\} = -\frac{1}{2}\,\Theta^{\mu\alpha}\Delta(\sigma+\bar{\sigma})\,, \tag{50}$$

$$\{\theta^\alpha(\sigma)\,, \bar{\theta}^\beta(\bar{\sigma})\} = \frac{1}{4}\,\Theta^{\alpha\beta}\Delta(\sigma+\bar{\sigma})\,, \tag{51}$$

where $\Delta(\sigma+\bar{\sigma})$ is different from zero only at string endpoints. We can conclude that interior of the string is commutative, while string endpoints are noncommutative.

In the case when the same chirality gravitinos ψ^α_μ and $\bar{\psi}^\alpha_\mu$ are equal, it follows that $\psi^\alpha_{-\mu} = 0$ and $(\Psi_{eff})^\alpha_\mu = \psi^\alpha_\mu$, so that we reproduct the results of Ref.[2].

5. Effective theory - type I superstring

To obtain an effective Hamiltonian we have to put the solutions of the constraints into the expression for canonical Hamiltonian. First, we introduce effective current

$$\tilde{I}_{\pm\mu} = p_\mu \pm \kappa G^{eff}_{\mu\nu} q'^\nu - 2(\Psi_{eff})^\alpha_\mu p_\alpha + 2\Pi_{\mp\mu}{}^\nu \psi^\alpha_{-\nu}(p_\alpha \pm \tilde{p}_\alpha)\,, \tag{52}$$

and correlate it with the currents $I_{\pm\mu}$ (5)

$$I_{\pm\mu} = \pm 2(\Pi_\pm G^{-1}_{eff})_\mu{}^\nu \tilde{I}_{\pm\nu}\,. \tag{53}$$

Using last relation and the solutions (43)-(45), the canonical Hamiltonian (4), turns to the effective one

$$\begin{aligned}\mathcal{H}^{eff}_c = &\tilde{t}_- - \tilde{t}_+ + \psi^\alpha_{-\mu}\left(\frac{1}{\kappa}G^{\mu\nu}_{eff}\tilde{p}_\alpha - 2\Theta^{\mu\nu}p_\alpha\right)p_\nu + \left(\tilde{\xi}'^\alpha + \psi^\alpha_{-\mu}q'^\mu\right)p_\alpha \\ &+ \left(\xi'^\alpha + \psi^\alpha_{+\mu}q'^\mu\right)\tilde{p}_\alpha - \frac{1}{2\kappa}p_\alpha\left(F^{\alpha\beta}_s + 4\kappa\psi^\alpha_{-\mu}\Theta^{\mu\beta}\right)p_\beta \\ &+ \frac{1}{2\kappa}\tilde{p}_\alpha F^{\alpha\beta}_s\tilde{p}_\beta + \frac{2}{\kappa}\tilde{p}_\alpha\psi^\alpha_{-\mu}(G^{-1}_{eff})^{\mu\nu}(\Psi_{eff})^\beta_\nu p_\beta\,,\end{aligned} \tag{54}$$

and expression $t_\pm$ defined in Eq.(5) turns to

$$\tilde{t}_\pm = \mp\frac{1}{4\kappa}(G^{-1}_{eff})^{\mu\nu}\tilde{I}_{\pm\mu}\tilde{I}_{\pm\nu}\,. \tag{55}$$

The effective Lagrangian

$$\mathcal{L}^{eff} = \dot{q}^\mu p_\mu + \dot{\xi}^\alpha p_\alpha + \dot{\tilde{\xi}}^\alpha\tilde{p}_\alpha - \tilde{\mathcal{H}}_c\,, \tag{56}$$

on the equation of motion for momentum p_μ obtains the form

$$\begin{aligned}\mathcal{L}^{eff} = &\frac{\kappa}{2}G^{eff}_{\mu\nu}\eta^{ab}\partial_a q^\mu\partial_b q^\nu + \left[\dot{\xi}^\alpha - \tilde{\xi}'^\alpha + 2(\Psi_{eff})^\alpha_\mu\dot{q}^\mu\right]p_\alpha r \\ &+ \left[\dot{\tilde{\xi}}^\alpha - \xi'^\alpha - 2(\Psi_{eff})^\alpha_\mu q'^\mu\right]\tilde{p}_\alpha + \frac{1}{2\kappa}p_\alpha F^{\alpha\beta}_{eff}p_\beta - \frac{1}{2\kappa}\tilde{p}_\alpha F^{\alpha\beta}_{eff}\tilde{p}_\beta\,,\end{aligned} \tag{57}$$

where effective background fields $G^{eff}_{\mu\nu}$, $(\Psi_{eff})^\alpha_\mu$ and $F^{\alpha\beta}_{eff}$ are defined in Eq.(39). Note that all terms in $\mathcal{L}^{eff}$ are symmetric under σ-parity.

Let us express original Lagrangian (1) in terms of variables with well defined σ-parity q^μ, ξ^α, $\tilde{\xi}^\alpha$, p_α and $\tilde{p}_\alpha$. After substitution initial variables with momenta independent parts of theirs solutions $x^\mu \to q^\mu$, $\theta^\alpha \to \eta^\alpha = \frac{1}{2}(\xi^\alpha +$

$\tilde{\xi}^\alpha$), $\bar{\theta}^\alpha \to \tilde{\eta}^\alpha = \frac{1}{2}(\xi^\alpha - \tilde{\xi}^\alpha)$ and fermionic momenta with their solutions $\pi_\alpha \to p_\alpha + \tilde{p}_\alpha$, $\bar{\pi}_\alpha \to p_\alpha - \tilde{p}_\alpha$, we have

$$\begin{aligned}
\mathcal{L} \to & \frac{\kappa}{2} G_{\mu\nu}\eta^{ab}\partial_a q^\mu \partial_b q^\nu + \kappa\varepsilon^{ab}B_{\mu\nu}\partial_a q^\mu \partial_b q^\nu \\
& + (\dot{\xi}^\alpha - \tilde{\xi}'^\alpha + \psi^\alpha_{+\mu}\dot{q}^\mu - \psi^\alpha_{-\mu}q'^\mu)p_\alpha + (\dot{\tilde{\xi}}^\alpha - \xi'^\alpha - \psi^\alpha_{+\mu}q'^\mu + \psi^\alpha_{-\mu}\dot{q}^\mu)\tilde{p}_\alpha\,, \\
& + \frac{1}{2\kappa}p_\alpha F_a^{\alpha\beta}p_\beta - \frac{1}{2\kappa}\tilde{p}_\alpha F_a^{\alpha\beta}\tilde{p}_\beta - \frac{1}{2\kappa}p_\alpha F_s^{\alpha\beta}\tilde{p}_\beta + \frac{1}{2\kappa}\tilde{p}_\alpha F_s^{\alpha\beta}p_\beta\,. \qquad (58)
\end{aligned}$$

Comparing this Lagrangian with effective one (57) we can conclude that $\mathcal{L} \to \mathcal{L}^{eff}$ after substitution

$$\begin{aligned}
& G_{\mu\nu} \to G^{eff}_{\mu\nu}\,, \quad \psi^\alpha_{+\mu} \to 2(\Psi_{eff})^\alpha_\mu\,, \quad F_a^{\alpha\beta} \to F^{\alpha\beta}_{eff}\,, \\
& B_{\mu\nu} \to 0\,, \qquad\quad \psi^\alpha_{-\mu} \to 0\,, \qquad\qquad F_s^{\alpha\beta} \to 0\,. \qquad (59)
\end{aligned}$$

Note that the terms which are not invariant under Ω transformation, $\varepsilon^{ab}B_{\mu\nu}\partial_a q^\mu\partial_b q^\nu$, $\psi^\alpha_{-\mu}q'^\mu p_\alpha$, $\psi^\alpha_{-\mu}\dot{q}^\mu\tilde{p}_\alpha$, $p_\alpha F_s^{\alpha\beta}\tilde{p}_\beta$ and $\tilde{p}_\alpha F_s^{\alpha\beta}p_\beta$ should disappear in effective theory, which produce the second line.

Consequently, in effective theory only Ω symmetric parts survive: graviton $G_{\mu\nu}$ in the NS-NS sector, gravitino $\psi^\alpha_{+\mu}$ in NS-R sector and field strength $F_a^{\alpha\beta}$ in R-R sector. As Ω symmetric part of the type IIB theory, effective theory corresponds to the type I superstring theory, but with improved background fields. As a consequence of boundary conditions at $\sigma = \pi$, this is a closed string theory.

6. Concluding remarks

In this paper we considered pure spinor formulation of the type IIB superstring introduced in Ref.[2]. We derived boundary conditions from the requirement that Hamiltonian, as time translation generator, has well defined functional derivatives in supercoordinates and their canonically conjugated supermomenta [8]. For bosonic coordinates x^μ we chose Neumann boundary conditions, while for same chirality fermionic coordinates θ^α and $\bar{\theta}^\alpha$ we chose that they are equal at the string endpoints, $(\theta^\alpha - \bar{\theta}^\alpha)|_0^\pi = 0$. Consequently, the corresponding fermionic canonical momenta satisfy the boundary conditions, $(\pi_\alpha - \bar{\pi}_\alpha)|_0^\pi = 0$.

All boundary conditions at string endpoints we treated as canonical constraints. Applying Dirac consistency procedure they produced an infinite set of constraints, which can be rewritten in the σ-dependent form. All these constraints, originating from boundary conditions, are of the second class and we can solve them. We obtained the expressions for supercoordinates x^μ, θ^α and $\bar{\theta}^\alpha$ in terms of effective ones q^μ, ξ^α and $\tilde{\xi}^\alpha$.

The theory obtained from initial IIB superstring theory on the solution of constraints we will call effective theory. Effective Lagrangian is described by effective supercoordinates q^μ, η^α and $\bar\eta^\alpha$. It is symmetric under orientfold projection Ω and consequently, it corresponds to the type I superstring theory. From boundary conditions it follows that effective theory is closed string one. This effective, type I, superstring propagates in effective background obtained from the type IIB one $G_{\mu\nu}$, $B_{\mu\nu}$, ψ^α_μ, $\bar\psi^\alpha_\mu$ and $F^{\alpha\beta}$ by substitution

$$\begin{aligned}
G_{\mu\nu} &\to G^{eff}_{\mu\nu} = G_{\mu\nu} - 4B_{\mu\rho}G^{\rho\sigma}B_{\sigma\nu}\,, & B_{\mu\nu} &\to 0\,,\\
\frac{1}{2}\psi^\alpha_{+\mu} &\to (\Psi_{eff})^\alpha_\mu = \frac{1}{2}\psi^\alpha_{+\mu} + B_{\mu\rho}G^{\rho\nu}\psi^\alpha_{-\nu}\,, & \psi^\alpha_{-\mu} &\to 0\,,\\
F_{\mu\nu\rho} &\to F^{eff}_{\mu\nu\rho} = F_{\mu\nu\rho} - \frac{1}{D}G^{\varepsilon\eta}\psi_{-\varepsilon}\Gamma_{[\mu\nu\rho]}\psi_{-\eta}\,, & F_\mu &\to 0\,,\ F_{\mu\nu\rho\sigma\varepsilon} \to 0\,,
\end{aligned} \tag{60}$$

where for R-R sector we used expressions in terms of antisymmetric tensors instead of bispinor. The first terms in effective backgrounds are just standard Ω projections (unoriented part of IIB superstring theory) and second parts are our improvements.

So, the type I superstring can not recognize explicitly oriented parts of IIB theory: NS-NS antisymmetric field $B_{\mu\nu}$, difference between two gravitinos $\psi^\alpha_{-\mu}$ and R-R fields: scalar C_0 and four rank antisymmetric tensor $C_{\mu\nu\rho\sigma}$, but it can see $B_{\mu\nu}$ and $\psi^\alpha_{-\mu}$ implicitly through effective backgrounds $G^{eff}_{\mu\nu}$, $(\Psi_{eff})^\alpha_\mu$ and $F^{eff}_{\mu\nu\rho}$.

On the solutions of the boundary conditions original string variables depend both on effective coordinates q^μ and ξ^α symmetric under σ-parity, and their canonically conjugated momenta p_μ and p_α. That property is a source of noncommutativity of initial supercoordinates. They do not depend on momentum $\tilde p_\alpha$, canonically conjugated to $\tilde\xi^\alpha$, so that this part, antisymmetric under σ-parity, does not contribute to noncommutativity relations. Also, the statement that field $B_{\mu\nu}$ is origin of space-time noncommutativity, we can generalize to supersymmetry statement that anyone of the nontrivial fields, $B_{\mu\nu}$ or $\psi^\alpha_{-\mu}$, are origins of noncommutativity between x^μ and θ^α, and anyone of the nontrivial fields $B_{\mu\nu}$, $\psi^\alpha_{-\mu}$ or $F^{\alpha\beta}_s$, are origins of noncommutativity between θ^α and θ^β.

In the case when the same chirality gravitinos ψ^α_μ and $\bar\psi^\alpha_\mu$ are equal, then $\psi^\alpha_{-\mu} = 0$ and $(\Psi_{eff})^\alpha_\mu = \psi^\alpha_\mu$ and we reproduct the results of Ref.[2].

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