String Field Theory and Cosmological Signature of Tachyon Condensation*

Alexey S. Koshelev†
Theoretische Natuurkunde, Vrije Universiteit Brussel, and
The International Solvay Institutes,
Pleinlaan 2, B-1050 Brussels, BELGIUM

Abstract

These are two lectures given by the author at "MUPh 2008" school in Belgrade. In the first lecture we are trying to give a consistent and concise introduction to the String Field Theory. We start with basic aspects of the Conformal Field Theory and then show how its machinery can be applied to the String Field Theory calculations. In the second lecture we consider the cosmological applications of the rolling tachyon coupled with the dilaton dynamics at the late times, especially to the Dark Energy problem.

1. Introduction.

In these lectures based on [1] (see also references therein) we present the most recent developments in the application of String Field Theory (SFT) motivated non-local scalar field models to cosmology. This is an extended version of the two lectures given by the author at "MUPh 2008" school in Belgrade.

The main motivation for the undertaken investigation is a wish to connect on one side the contemporary cosmological observational data [2] which strongly support that the present Universe exhibits an accelerated expansion providing thereby an evidence for a dominating Dark Energy (DE) component and on the other side the most promising candidate for the unified theory of interaction, the string theory and specifically the SFT.

Recent results of WMAP [3] together with the data on Ia supernovae give the following bounds for the DE state parameter $w_{\mathrm{DE}} = -1^{+0.14}_{-0.11}$ or without an a priori assumption that the Universe is flat and together with the data

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† e-mail address: koshelev@tena4.vub.ac.be

on large-scale structure and supernovae $w_{\rm DE} = -1.06^{+0.13}_{-0.08}$. This range of w includes quintessence models, $w > -1$, containing an extra light scalar field which is not in the Standard Model set of fields, the cosmological constant, $w = -1$, and "phantom" models, $w < -1$, which can be described by a scalar field with a ghost (phantom) kinetic term. In this case all natural energy conditions are violated and there are problems of instability both at the classical and quantum levels. Models with a crossing of the $w = -1$ barrier are also a subject of recent studies. Simplest ones include two scalar fields (one phantom and one usual field). General κ-essence models can have both $w < -1$ and $w \geqslant -1$ but a dynamical transition between these domains is forbidden under general assumptions.

Some projects are directly aimed at exploring whether w varies with the time or is a constant (see [3] and refs. therein). Varying w obviously corresponds to a dynamical model of the DE which generally speaking includes a scalar field. Modified models of General Relativity also generate an effective scalar field. Other DE models based on brane-world scenarios are presented in. A comprehensive review [4] and references therein may provide the reader with a more detailed discussion of the DE dynamics.

In the present lectures we summarize the research along the lines of [1] and present the cosmological model coming from the open String Field Theory (SFT) tachyon dynamics (see also [5]), namely from the cubic SFT formulation [6] of the open fermionic NSR string with the GSO$-$ sector [7] (see [8] for a review). These models address the issue of DE and its possible dynamics.

In the first lecture we are trying to give a consistent and concise introduction to the SFT. We start with basic aspects of the Conformal Field Theory (CFT) and then show how its machinery can be applied to the SFT calculations. In the second lecture we consider the cosmological applications of the rolling tachyon dynamics at the late times (or, in other words, at present).

2. Lecture I. Basics of SFT.

2.1. CFT

In the sequel we mainly follow the notations from the Polchinski textbook on Superstrings [9] (see also [10]). We use Minkowskian signature $\eta_{\mu\nu} = \mathrm{diag}(-,+,+,+,\dots,+)$ and work in $1+d$-dimensional space. On the world-sheet whereas we use the Euclidean continuation and work on the conformal plane $z = e^{\tau+i\sigma}$ with σ going from 0 to π.

The world-sheet action for the superstring has the form

$$S = \frac{1}{2\pi\alpha'}\int d^2z\left(\partial X^\mu\bar{\partial}X_\mu + \frac{\alpha'}{2}\left(\psi^\mu\bar{\partial}\psi_\mu + \tilde{\psi}^\mu\partial\tilde{\psi}_\mu\right)\right) \tag{1}$$

supplemented by the world-sheet action for ghost fields

$$S_{\rm ghosts} = \frac{1}{2\pi}\int d^2z\left(b\bar{\partial}c + \beta\bar{\partial}\gamma + \tilde{b}\partial\tilde{c} + \tilde{\beta}\partial\tilde{\gamma}\right) \tag{2}$$

The following Operator Product Expansions (OPEs) are easily obtained from actions (1,2) having Neumann boundary conditions in mind

$$\begin{aligned} X^\mu(z,\bar z)X^\nu(w,\bar w) &\sim -\frac{\alpha'}{2}\eta^{\mu\nu}\left(\log|z-w|+\log|\bar z-w|\right)\\ X^\mu_{\rm Left}(z)X^\nu_{\rm Left}(w) &\sim -\frac{\alpha'}{2}\eta^{\mu\nu}\log(z-w)\\ \partial X^\mu(z)\partial X^\nu(w) &\sim -\frac{\alpha'}{2}\frac{\eta^{\mu\nu}}{(z-w)^2}\\ \psi^\mu(z)\psi^\nu(w) &\sim \frac{\eta^{\mu\nu}}{z-w}\\ b(z)c(w)\sim c(z)b(w) &\sim \frac{1}{z-w}\\ \beta(z)\gamma(w)\sim -\gamma(z)\beta(w) &\sim -\frac{1}{z-w} \end{aligned}$$

Hereafter we will use $X_{\rm Left}(z)$ and omit for simplicity the subscript "Left" whenever this does not lead to ambiguity. Thus momentum dependent vertex $e^{ikX(z,\bar z)}$ which is $e^{2ikX_{\rm Left}(z)}$ now will be denoted as $e^{2ikX(z)}$. OPE of the product of several exponents of X is

$$\prod_j e^{2ik_jX(z_j)} = \prod_{m<n}(z_m-z_n)^{2\alpha' k_m k_n}e^{2i\sum_j k_jX(z_j)}$$

Hereafter exponents of conformal fields are assumed to be normal ordered (to be discussed below). The superghosts β,γ can (and should) be bosonized as follows

$$\beta(z)=e^{-\varphi}\partial\xi(z),\quad \gamma(z)=\eta e^{\varphi}(z)$$

OPEs for η,ξ and φ have the form

$$\begin{aligned} \eta(z)\xi(w)\sim\xi(z)\eta(w) &\sim \frac{1}{z-w}\\ \varphi(z)\varphi(w) &\sim -\log(z-w) \end{aligned}$$

Note, that exponents $e^{n\varphi}$ behave like bosons for even n and like fermions for odd n in permutations. For our purposes we consider only integer n. OPE of the product of several exponents of φ is

$$\prod_i e^{n_i\varphi(z_i)} = \prod_{i<j}(z_i-z_j)^{-n_in_j}e^{\sum_i n_i\varphi(z_i)}$$

Stress tensors for matter and ghost fields are readily obtained from (1) and

(2) to be

$$
\begin{aligned}
&T^X = -\frac{1}{\alpha'} : \partial X \partial X : \quad T^\psi = -\frac{1}{2} : \psi \partial \psi : \\
&T^{bc} = -2 : b \partial c : - : (\partial b) c : \quad T^{\beta\gamma} =: (\partial \beta) \gamma : -\frac{3}{2} : \partial(\beta\gamma) : \\
&T^{\xi\eta} = - : \eta \partial \xi : \quad T^\varphi = -\frac{1}{2} : \partial\varphi\partial\varphi : - \partial^2 \varphi
\end{aligned}
$$

and the matter supercurrent is

$$
G = i\sqrt{\frac{2}{\alpha'}} : \psi \partial X :
$$

with a contraction with respect to a vector index assumed. $: \cdots :$ is the normal ordering to be discussed below. Here we already assumed particular values for weights of ghost fields. In fact the action must be conformally invariant. This assumption gives exact weights for ∂X^μ and Ψ^μ. However, for ghost fields, say for (b, c)-system, one can take arbitrary weights λ and $1 - \lambda$ respectively (and analogously for (β, γ)-system). On the other hand anomaly free condition fixes weights of ghost fields. In other words, absence of the central charge and nilpotency of the BRST charge in the full theory imply exact weights for all ghost fields.

The BRST charge is

$$
Q = \oint \frac{dz}{2\pi i} \left(: c \left(T^X + T^\Psi + T^{\xi\eta} + T^\varphi \right) + bc\partial c - b\eta\partial\eta e^{2\varphi} + \eta e^\varphi G : \right) \quad (3)
$$

Let us explore a bit more some issues of the CFT. The holomorphic conformal field $\phi(z)$ is called *primary* field of the weight h provided it transforms under conformal transformation $f(z)$ as

$$
f \circ \phi(z) = (f'(z))^h \phi(f(z))
$$

Hereafter if something else not specified symbols ∂ and $'$ mean derivatives with respect to function argument.

The normal ordered product $: \cdots :$ (or simply normal product) introduced above is defined as a finite part of the OPE in the limit of coincident points while the singular part of the OPE (in the limit of coincident points) is denoted as $\langle \dots \rangle$, i.e.

$$
\phi_1(z_1)\phi_2(z_2) =: \phi_1(z_1)\phi_2(z_2) : + \langle \phi_1(z_1)\phi_2(z_2) \rangle + o(z_1 - z_2) \quad (4)
$$

The singular part may contain conformal fields as an OPE of the exponent of X^μ and ∂X^μ. In order to define a normal ordered product of more than two operators one has to apply the definition (4) recursively for all possible permutations. Such a construction is the Wick theorem in CFT.

Using this we can calculate conformal transformations of normal products of conformal fields. To do this first we have to separate points using inverse form of (4) (again recursively, if needed). This gives us a right to transform the fields independently while they are in different points. Second we have to apply conformal transformation to all fields. Third, we have to subtract singularities in new coordinates. Forth, we have to take a limit of coincident initial points. Let us give a short sample calculation.

$$\begin{aligned}
& f\circ :\partial X^{\mu} e^{2ikX} : (z) = \\
& = \lim_{\substack{z_1\to z \\ z_2\to z}} \left(f\circ \left(\partial X^{\mu}(z_1) e^{2ikX}(z_2) - \langle \partial X^{\mu}(z_1) e^{2ikX}(z_2)\rangle \right)\right) = \\
& = (f'(z))^{\alpha' k^2+1} \left(:\partial X^{\mu} e^{2ikX} : (f(z)) - i\alpha' k^{\mu} \frac{f''(z)}{2(f'(z))^2} e^{2ikX}(f(z)) \right)
\end{aligned}$$

For a given primary field $\phi(z)$ the OPE with the stress tensor is

$$T(w)\phi(z) \sim \frac{h}{(w-z)^2} + \frac{1}{w-z}\partial_z\phi(z)$$

where $\sim$ means that we neglect all regular with respect to $w-z$ terms. Further one can expand the field $\phi(z)$ as

$$\phi(z) = \sum_n \phi_n z^{-n-h}$$

Contour integration gives

$$\oint \frac{dz}{2\pi i}\phi(z) z^{n+h-1} = \phi_n$$

If we assume our conformal fields to be analytic then we find the vacuum

$$\phi_n|0\rangle = 0 \text{ for } n \geq 1-h$$

This vacuum is called $SL(2,R)$-*invariant* vacuum.

Skipping the details we arrive to the fact that $SL(2,R)$-invariant vacuum for the (β,γ)-system is determined by the condition

$$\beta_r|0\rangle = 0 \text{ for } r > -\frac{3}{2}, \quad \gamma_r|0\rangle = 0 \text{ for } r > \frac{1}{2}$$

Recalling that γ is bosonic field we see that the spectrum is unbounded from below (using that $[L_0,\phi_n] = -n\phi_n$ for a primary field ϕ and L_0 being zero Virasoro operator). However one cannot pass to another vacuum by means of polynomial of β and γ. The right way is to bosonize superghosts. Then one finds that the state

$$e^{-\varphi}|0\rangle$$

is annihilated by all operators β and γ with positive indexes. The point is that now we have obtained completely different representation of the states. In general such representations are called *pictures*.

Introducing ghost and picture currents

$$j_{\mathrm{gh}} = -:bc:-:\xi\eta: \quad j_{\mathrm{p}} =: \xi\eta : -\partial\varphi$$

and skipping all further details we can summarize the result of our consideration in Table 1. Additionally we have to note that the field φ has a

Field	Weight	Ghost #	Picture #
∂X^μ	1	0	0
e^{2ikX}	$\alpha' k^2$	0	0
Ψ^μ	$1/2$	0	0
b	2	-1	0
c	-1	1	0
β	$3/2$	-1	0
γ	$-1/2$	1	0
η	1	1	1
ξ	0	-1	-1
$\partial\varphi$	1	0	0
$e^{n\varphi}$	$-\frac{1}{2}n(n+2)$	0	n

Table 1: Properties of fields

background charge in its free action and this leads to unusual transformation properties. Namely

$$f \circ \varphi(z) = \varphi(f(z)) - \log f'$$

Also there is no field ξ in the play, only $\partial\xi$ and its descendants. This is called a small Hilbert space.

It is a matter of simple calculations to check that the fields introduced in such a way do describe well defined anomaly free theory. Moreover one finds that the correlation function will be non-zero once we have 3 extra c fields and the total picture of the expression is -2. The correlation function for c fields is

$$\langle c(z_1)c(z_2)c(z_3)\rangle = (z_1 - z_2)(z_2 - z_3)(z_1 - z_3)$$

The normalization we use is

$$\langle c\partial c\partial^2 c e^{-2\varphi} e^{2ikX}\rangle = -2(2\pi)^{d+1}\delta^{d+1}(k) \tag{5}$$

Two useful operators should be introduced here

$$X = c\partial\xi + e^{\varphi}G + e^{2\varphi}b\partial\eta + \partial(e^{2\varphi}b\eta), \quad Y = c\partial\xi e^{-2\varphi}$$

which raise and lower picture by 1 respectively. One can straightforwardly check that normalization of above operators is correct and

$$X(z)Y(w) \cong Y(z)X(w) \cong 1$$

where $\cong$ means nonzero part of the OPE in coincident points.

2.2. SFT

The action for the SuperString Field Theory in the Neveu-Schwarz sector has the form

$$S = \frac{1}{g_o^2}\left(\frac{1}{2\alpha'}\langle\langle Y_{-2}|\mathcal{A}, Q\mathcal{A}\rangle\rangle + \frac{1}{3}\langle\langle Y_{-2}|\mathcal{A}, \mathcal{A}, \mathcal{A}\rangle\rangle\right) \tag{6}$$

Here g_o is a coupling constant, $\mathcal{A}$ is the String Field, Y_{-2} is the double step picture changing operator and $\langle\langle \dots \rangle\rangle$ denote a special correlation function of the CFT.

The string field is an infinite sum of the following form

$$\mathcal{A}(z) = \int d^D k \sum_I \phi_I(k)\Phi_I(k,z) \tag{7}$$

where $\phi_I(k)$ is a space-time field, $\Phi_I(k,z)$ is a vertex operator corresponding to that space-time field and I is a multiindex. Working at classical level one can restrict the consideration to the case that the ghost number is equal to 1 and the picture number is equal to 0. If this is the case than weights of vertex operators are bounded from below (in general for any fixed values of ghost and picture numbers the minimal weight exists). So, while we work at classical level we have minimal weight state and in our case the minimal weight is -1. The corresponding vertex operator is $c(z)$. Of course, we do not take into account momentum-dependent multiplier e^{2ikX} which gives momentum dependent weight $\alpha' k^2$, i.e. we consider static configuration. Thus we can naturally arrange summands in the sum (7) starting from the minimal weight. Further we can truncate the series taking into account only terms with the weight less or equal some given number. This is the so called level truncation scheme.

On the conformal language a propagating string is an expanding semicircle in the upper half-plane (UHP) with the center in the origin. The proper time τ is an absolute value and string parameter σ is a phase of the complex coordinate z. Thus at zero proper time the string is unit semicircle and point i is the midpoint. Note, that from the beginning all vertex operators have been inserted at points 0 of their corresponding strings. Assuming that interaction takes place at zero time we have to glue our strings in accord with Witten's original proposal formulated without conformal language [11]. Namely, we have to glue second half of the first string and first half of the second string taking opposite directions how the parameter σ changes and so on for three-string interaction. The midpoint in this prescription is mapped to the same point for all strings. Double brackets $\langle\!\langle \ldots \rangle\!\rangle$ denote such gluing. The kinetic term can be expressed in the form

$$\langle\!\langle Y_{-2}|\mathcal{A}, Q\mathcal{A}\rangle\!\rangle = \lim_{w\to 0}\langle Y(i), Y(-i), \mathrm{bpz}\circ\mathcal{A}(w), Q\mathcal{A}(0)\rangle$$

where $\mathrm{bpz}(z) = -1/z$. For the interaction the first step is to make gluing onto the unit disk. At this step all original zeros are on the disk boundary and all midpoints are mapped to the origin. Then we can map this disk to whatever we want because of $SL(2,R)$ invariance. A natural choice is to make a mapping to the UHP such that original zeros go to some points on the real axis, say 1, 0, -1 for the interaction. There is one subtlety here. Working with GSO$-$ sector we have half-integer weights (Indeed, in the NS sector fields with integer weights are GSO+ and with half-integer weights are GSO$-$). This means that we have to work with half-integer powers when calculate conformal transformation. Roughly speaking we have to specify which root is of relevance for our calculations. To do this we state that the argument of complex numbers goes from 0 to 2π. Then working on the disk we use the rule

$$\left(|z|e^{i\arg(z)}\right)^{1/2} = \left||z|^{1/2}\right| e^{i\arg(z)/2}$$

We have to keep this rule, i.e. to have clear understanding how to choose roots after mapping to the UHP. The result is

$$f_k(z) = f_{\mathrm{UHP}}\circ f_{\mathrm{disk}}(z)$$

$$f_{\mathrm{disk}}(z) = \left(\frac{1+iz}{1-iz}\right)^{2/3} e^{-\frac{2}{3}\pi i(k-2)},\quad f_{\mathrm{UHP}}(z) = -\frac{i}{\sqrt{3}}\frac{z-1}{z+1},\quad k=1,2,3$$

These maps are very convenient for computer calculations because first derivatives for all k are positive and to keep trace to our rule on the disk we have to take positive branch of the square root. For any k the midpoint i is mapped to $i/\sqrt{3}$. Therefore, for the interaction we have

$$\langle\!\langle Y_{-2}|\mathcal{A},\mathcal{A},\mathcal{A}\rangle\!\rangle = \langle Y(i/\sqrt{3}), Y(-i/\sqrt{3}), f_1\circ\mathcal{A}(0), f_2\circ\mathcal{A}(0), f_3\circ\mathcal{A}(0)\rangle$$

To incorporate both GSO+ and GSO$-$ sectors one has to understand all ingredients to be tensor products of themselves and 2×2 matrices. Also, as usually, an operator $\frac{1}{2}\mathrm{tr}$ is inserted in front of the action. Namely,

$$\mathcal{A} = \mathcal{A}_+\otimes\sigma_1 + i\mathcal{A}_-\otimes\sigma_2,\quad Y_{-2} = Y_{-2}\otimes\sigma_3,\quad Q = Q\otimes\sigma_3$$

where σ_i are Pauli matrices. The gauge transformation leaving the action (6) invariant is

$$\delta\mathcal{A} = Q\Lambda + \alpha'[A,\Lambda]_*$$

where Λ is gauge parameter and $*$ is Witten's string product. Λ can be understood as a string field with ghost number 0 and picture number 0.

2.3. Level approximation and States

As it was described above we know the level truncation scheme. Here I'll not discuss a validity of this approximation. In fact this is the only method we can do any computations in SFT. Let us recall that in this approach we truncate the series of the string field at some mass level M and perform calculations using such a truncated string field. To do this we have at least know that our series is bounded from below, i.e. we have lowest mass state. This is true if the ghost number and picture number are fixed. Let us explain this in more details for our case ghost# = 1 and picture# = 0. Let us denote by B, C, H, Ξ number of corresponding conformal fields in a given state and let us use the number q as the exponent factor of the exponent of φ and q is integer. Than using values from the Table 1 we have the following system

$$C - B + \mathrm{H} - \Xi = 1, \qquad q + \Xi - \mathrm{H} = 0$$

Also note that the statement "there are B fields b" together with the minimal weight requirement means the following expression

$$b(z)\partial b(z)\dots\partial^{B-1}b(z)$$

Indeed, we can not have two fields b with equal number of derivatives because of nilpotency. Also each derivative increase the weight by 1 and expression $b\partial^2 b$ has higher weight than $b\partial b$. Thus the expression above has the minimal weight for the given number B of fields b. The weight for such a combination is

$$\sum_{k=1}^{B}(1+k) = B + \frac{B}{2}(B+1)$$

while the weight of the field b is 2. Now it is convenient to consider q as a parameter.

For $q > 1$ we have to have at least q fields η, $q-1$ fields b and 0 fields ξ and c. Doing this we saturate equations for ghost and picture numbers. Other (matter) fields do not enter these equations. Moreover matter fields have positive weights and to have minimal weight state we do not consider them. Combinations of minimal weights for η and b have weights

$$h_\eta = \sum_{k=1}^{q} k = \frac{q}{2}(q+1), \qquad h_b = \sum_{k=1}^{q-1}(1+k) = (q-1) + \frac{q}{2}(q-1)$$

respectively. Recalling that the weight of the exponent of φ for a given q is $-\frac{1}{2}q(q+2)$ we have for the minimal weight

$$h_{\min} = -\frac{1}{2}q(q+2) + \frac{q}{2}(q+1) + (q-1) + \frac{q}{2}(q-1) = \frac{q^2}{2} - 1$$

Exploring in the same way cases $q = 1, q = 0$ and $q < 0$ we come to the following result

$$\begin{array}{llcl}
\text{For } q > 0 & \text{at least} & \begin{array}{r} q \text{ fields } \eta \\ q-1 \text{ fields } b \end{array} & h_{\min} = \dfrac{q^2}{2} - 1 \qquad (8) \\
\text{For } q \leq 0 & \text{at least} & \begin{array}{r} -q \text{ fields } \xi \\ -q+1 \text{ fields } c \end{array} & h_{\min} = \dfrac{q^2}{2} - q - 1 \qquad (9)
\end{array}$$

At this point it is evident that the weight is bounded from below and the lowest state is $c(z)$ with $q = 0$ which has weight -1.

The next step is to construct all states for a given weight $|h\rangle$. Here we will explain an algorithm which gives all states in a systematic way. First, we restrict possible values of q for which states with given ghost and picture numbers exist. Second, we unambiguously construct a state with a minimal weight for the given q $|h_q\rangle$. Third, we raise up the weight of the state $|h_q\rangle$ up to the weight h in question if $h_q < h$. The later means that we multiply the state $|h_q\rangle$ by other fields or apply a derivative in order to achieve the weight h. There are three subtleties at this stage. First, we can multiply only by pairs of fields (b, c) or (ξ, η) because otherwise we will violate ghost or picture numbers. Second, we cannot multiply by bc if our state already has c while c is nilpotent. So, we have to multiply by $b\partial c$. Also, applying a partial derivative to, say $c\partial c$ combination we will have only one new state which is $c\partial^2 c$ while $\partial c\partial c \equiv 0$. Third, if the difference $h - h_q$ is half-integer we have to use at least one field ψ^μ or its derivative while this is the only field of half-integer weight. Using all the above we can construct all the states $|h\rangle$. To do this we have to search for all possible different ways to raise up the weight from h_q to h. This can be done algorithmically.

For example, let us construct all the states of the weight $h = 0$. First, we see that only $q = 0, 1$ are of interest. For $q = 0$ the lowest weight state is simply c which has the weight $h_0 = -1$. To achieve the weight 0 we have to raise up h_0 by 1. What are the possibilities? They are $\partial X^\mu, \psi^\mu\psi^\nu, bc, \partial\varphi$. There is no combination consisting of (ξ, η) while in the small Hilbert space we have only $\partial\xi$ and the minimal possible weight is 2, namely, the pair $\partial\xi\eta$. Also the combination bc is irrelevant because c is nilpotent. Another way is to apply a partial derivative. Summing all up we have the following states

$$c\partial X^\mu, \quad c\psi^\mu\psi^\nu, \quad c\partial\varphi, \quad \partial c$$

$q = 1$ gives the minimal weight state ηe^φ with $h_1 = -1/2$. There is only possibility to construct a state of the weight 0 which is multiply ηe^φ by ψ^μ. So we have one more state of the weight 0

$$\eta e^\varphi \psi^\mu$$

One can check that these are all states of the weight 0.

Construction of all states of higher weights is straightforward but very tedious. However, it is a matter of simple exercise to implement this method in any programming language.

2.4. Evaluation of the action

It is natural to start from an evaluation of the kinetic term. A very important ingredient is action of the BRST charge Q on the string field $\mathcal{A}$. To do this we have to calculate

$$Q\mathcal{A}(0)=\frac{1}{2\pi i}\oint j_{BRST}(\zeta)d\zeta\mathcal{A}(0)=\operatorname*{Res}_{\zeta=0}j_{BRST}(\zeta)\mathcal{A}(0)$$

where the RHS of this expression contains OPE of the BRST current and the string field. The key point is that picking up a residue commutes with calculation of the correlation function. Thus one can first compute the correlation function corresponding to the kinetic term and then pick up a residue

$$\begin{aligned}\langle\langle Y_{-2}|\mathcal{A},Q\mathcal{A}\rangle\rangle&=\lim_{w\to0}\langle Y(i),Y(-i),\mathrm{bpz}\circ\mathcal{A}(w),Q\mathcal{A}(0)\rangle=\\&=\lim_{w\to0}\operatorname*{Res}_{\zeta=0}\langle Y(i),Y(-i),\mathrm{bpz}\circ\mathcal{A}(w),j_{BRST}(\zeta)\mathcal{A}(0)\rangle\end{aligned}$$

The following sequence of operations $(\langle\ldots\rangle,\mathrm{Res},\lim)$ is the most convenient for computer computations.

Computation of the correlation function is the most laborious step. In spite of the fact that all calculations are straightforward let me say few words about this. To compute the correlation function we have to calculate first the conformal transformation of the string field $\mathcal{A}$. One has to exercise an extreme care at this point because in most cases vertex operators are composite operators rather than primary fields. To be normal products of primary conformal fields or descendants they have not obvious conformal transformations. Also after transformation number of vertex operators may increase because of Leibniz rule. The next step which is actual calculation using OPE and normalization (5) is more straightforward but is not simple. The main difficulty is that one has to perform too many very simple operations. In fact an amount of work drastically increases if we take higher levels into account. This indicates that the proper way is to use a computer to perform actual calculations in SFT (or SuperSFT). Doing this we simply shield our work from accidental errors and misprints. Other steps, picking up a residue and taking a limit are evident.

Computation of the interaction is simpler because there are no limits, residues, etc.

Let us introduce a notion of *level* which is weight + 1. It looks a bit more convenient than the weight because it starts from 0 in the 0 picture. Usually if we do not state something else saying "truncation up to level n" means that we include all fields up to level n and calculate all interaction terms so the highest level term in the action might be $3n$.

Now we are ready to apply all the above to actual calculations.

2.5. Actual Calculations. Action (6) for levels 0, 1/2 (negative weights).

We have in the play the vertexes c, ηe^{φ} and $c\psi^{\mu}$ which form the string fields as

$$\begin{aligned}\mathcal{A}_{+}(z) &= \int d^D k u(k) c(z) e^{2ikX}(z)\\ \mathcal{A}_{-}(z) &= \int d^D k \left(\tau(k)\eta e^{\varphi}(z) + s_{\mu}(k) c\psi^{\mu}\right) e^{2ikX}(z)\end{aligned} \tag{10}$$

To calculate the action we have to calculate conformal transformation first and the corresponding expressions are

$$\begin{aligned}f\circ\mathcal{A}_{+}(z) &= \int d^D k \left(u(k)\frac{1}{f'(z)}c(f(z))\right)(f'(z))^{\alpha' k^2} e^{2ikX}(f(z))\\ f\circ\mathcal{A}_{-}(z) &= \int d^D k \frac{1}{\sqrt{f'(z)}}\left(\tau(k)\eta e^{\varphi}(f(z)) + s_{\mu} c\psi^{\mu}(f(z))\right)\\ &\quad\times (f'(z))^{\alpha' k^2} e^{2ikX}(f(z))\end{aligned}$$

Substituting string fields and their conformal transformations in the action and performing all the calculations after the Fourier transform

$$\varphi(x) = \int d^D k \varphi(k) e^{ikx}$$

we arrive to the following result

$$\begin{aligned}S_0 =& \frac{1}{g_o^2}\int d^D x \left(\frac{1}{2\alpha'}\left(\frac{1}{2}u^2(x) + \right.\right.\\ &\left.\left.+\frac{1}{2}\tau^2(x) - \frac{1}{2}s_{\mu}^2(x) + i\sqrt{2\alpha'}\partial_{\mu}s^{\mu}(x)\tau(x)\right) + \frac{9}{16}\tilde{\tau}^2(x)\tilde{u}(x)\right)\end{aligned} \tag{11}$$

with

$$\tilde{\varphi}(x) = e^{-\alpha' \log\gamma \partial^2}\varphi(x)$$

Substituting the EOM for s_{μ}

$$s_{\mu}(x) = -i\sqrt{2\alpha'}\partial_{\mu}\tau(x) \tag{12}$$

back into (11) we obtain an effective action for remaining fields $u(x)$ and $\tau(x)$ which is

$$S_{0,\text{eff}} = \frac{1}{g_o^2}\int d^D x \left(\frac{u^2(x)}{4\alpha'} - \frac{(\partial_{\mu}\tau(x))^2}{2} + \frac{\tau^2(x)}{4\alpha'} + \frac{9}{16}\tilde{\tau}^2(x)\tilde{u}(x)\right) \tag{13}$$

This is exactly the action studied in [12]. One can further exclude the field $u(x)$ since it is gaussian auxiliary field. We see that the action (13) exhibits expected behavior at tree level and free action for the tachyon is restored.

3. Lecture II. Tachyon Condensation and its cosmological signatures

3.1. Cosmological Rolling Tachyon

The open string tachyon when all massive states are integrated out by means of equations of motion acquires a non-trivial potential with a non-perturbative minimum. Rolling of the tachyon from the unstable perturbative extremum towards this minimum describes, according to the Sen's conjecture [13], a transition of an unstable D-brane to a true vacuum where no perturbative states of the open string are present. A solution corresponding to a true non-perturbative vacuum in the open fermionic NSR string with the GSO$-$ sector described by the modified cubic SFT has been found recently [14] and the Sen's conjecture has been proved.

Action (13) after an obvious rescaling of coordinates and fields and an adding of the Einstein gravity sector reads

$$S = \int d^D x\sqrt{-g}\left(\frac{R}{2\kappa^2} + \frac{1}{g_o^2}\left(-\frac{1}{2}g^{\mu\nu}\partial_\mu\tau\partial_\nu\tau + \frac{1}{2\alpha'}\tau^2 - \frac{1}{\alpha'}v(\tilde{\tau})\right)\right) \quad (14)$$

Here κ is the gravitational coupling constant $\kappa^2 = 8\pi G = \frac{1}{M_P^2}$ (these assignments are true in $D=4$), G is the Newton's constant, M_P is the Planck mass, α' is the string length squared, g is a metric, $\Box = D^\mu\partial_\mu$ and D_μ being a covariant derivative. Here g_o is the open string coupling, $[g_o]$ = length in $D=4$,

$$\tilde{\tau} = \mathcal{G}_o(\alpha'\Box)\tau,$$

and τ is the open string tachyon, it is dimensionless. $\mathcal{G}_o$ is supposed to be an analytic function of the argument. $v(\tilde{\tau})$ is an open string tachyon potential and it is supposed to be an analytic function in its argument. Factor $1/\alpha'$ in front of the tachyon potential looks unusual and can be easily removed by a rescaling of fields. For our purposes it is more convenient keeping all the fields dimensionless.

Tachyon dynamics in flat background was studied analytically and numerically in [12]. It was shown that there exist a solution interpolating between two symmetric vacua. The dynamics in a given curved background may be studied as well. But the most interesting is a dynamical gravity and in this case only numeric methods exist so far. It is still possible, however, considering an asymptotic behavior of the tachyon at late times.

We consider the tachyon near its true vacuum and linearize equations of motion using $\tau = \tau_0 + \zeta$. A linearized equation becomes

$$\Box\zeta + \frac{\zeta}{\alpha'} - \frac{v''(\tau_0)}{\alpha'}\tilde{\tilde{\zeta}} = 0. \quad (15)$$

A way of solving such an equation was developed in [1, 15]. The main idea is that an eigenfunction of the $\Box$ operator with an eigenvalue $\omega^2 = \bar{\omega}^2/\alpha'$

solves the latter equation if $\bar{\omega}^2$ solves the transcendental equation

$$\left(\bar{\omega}^2+1\right)\mathcal{G}_o^{-2}(\bar{\omega}^2)=v''(\tau_0). \tag{16}$$

Using a level truncated cubic fermionic string field theory we have $\mathcal{G}_o^{-2}(\bar{\omega}^2) = e^{\beta\bar{\omega}^2}$. A solution for $\bar{\omega}^2$ becomes

$$\bar{\omega}^2=\frac{W(\gamma)-\beta}{\beta},\quad \gamma=v''(\tau_0)\beta e^{\beta}, \tag{17}$$

where W is Lambert W-function which solves an equation $xe^x = y$ w.r.t. x. All $\bar{\omega}$, β and γ are dimensionless. Notice, that for $v''(\tau_0) = 1$ we have $\omega^2 = 0$ irrespectively of β.

In fact there are infinitely many branches of this function in a general situation. Zero branch is distinguished since it gives real values on the positive semi-axis. On the negative semi-axis a branch point of zero and -1 branches is located at $-1/e$ and $W_{(0)}(-1/e) = -1$ (hereafter a subscript (n) denotes the branch). For positive ϵ one finds that $W_{(0)}(-1/e+\epsilon)$ is real while $W_{(0)}(-1/e-\epsilon)$ is complex. In case of complex ω^2 one readily finds that $W_{(-1)}(-1/e-\epsilon) = W_{(0)}(-1/e-\epsilon)^*$. This provides a chance to write a real solution for τ as a linear combination of functions corresponding to $\omega^2_{(0)}$ and $\omega^2_{(-1)}$. Zero and -1 branches give smallest in absolute value roots and should be considered as a main contribution to a solution of our linearized equation.

Thus, if $\gamma \geq -1/e$ then ω^2 is real and is given by the zero branch of the Lambert function. A solution to equation (15) is given by a function which satisfies

$$\Box\zeta=\omega^2\zeta. \tag{18}$$

If $\gamma < -1/e$ then ω^2 is complex and is given by the zero and -1 branches of the Lambert function. A solution to equation (15) is given by a linear combination of functions which satisfy

$$\Box\zeta_{(0)}=\omega^2_{(0)}\zeta_{(0)},\quad \Box\zeta_{(-1)}=\omega^2_{(-1)}\zeta_{(-1)}. \tag{19}$$

Integration constants of solutions $\tau_{(0)}$ and $\tau_{(-1)}$ can be adjusted such that resulting $\zeta = \zeta_{(0)} + \zeta_{(-1)}$ is real. Alternatively, one can say that fluctuations are effectively described by two freely propagating fields with masses squared $-\omega^2_{(0)}$ and $-\omega^2_{(-1)}$. Although a quantity corresponding to the initial field ζ can be made real both effective scalar fields have complex masses squared. One can consider real linear combinations of the effective fields. In this case one field will be either a ghost or a tachyon. This is in agreement with the Ostrogradski analysis [16].

Thus, in order to analyze the dynamics of the tachyon at large times one has to solve equation (18) (or (19)) which for a constant Hubble parameter gives as a solution

$$\zeta = e^{-\frac{3}{2}Ht}\mathrm{Re}\left(\zeta_+ e^{\frac{1}{2}t\sqrt{9H^2-4\omega^2}} + \zeta_- e^{-\frac{1}{2}t\sqrt{9H^2-4\omega^2}}\right). \qquad (20)$$

Recall that ω^2 is given by equation (17) and is in general a complex number.

Having sketched the main line of solving linear non-local equations and before analyzing possible cosmological consequences we are going to add the dilaton field into the play.

3.2. SFT in Linear Dilaton Background

The worldsheet action for the open string in the linear dilaton background in the flat space-time is given by

$$S = \frac{1}{4\pi\alpha'}\int d\tau d\sigma\sqrt{h}\left(h^{\tau\sigma}\partial_\tau X^\mu \partial_\sigma X_\mu + \alpha' \mathcal{R} V_\mu \partial X^\mu + \text{fermions} + \text{ghosts}\right).$$

Here $\mathcal{R}$ is the two-dimensional curvature and V_μ comes from the linear dilaton $\Phi = 2V_\mu X^\mu$ playing a role of the background charge. As compared to the ordinary CFT the bosonic stress tensor and the matter supercurrent get modified and look like

$$T^X = -\frac{1}{\alpha'} :\partial X \partial X: + V\partial^2 X, \quad G = i\sqrt{\frac{\alpha'}{2}}(:\psi\partial X: - \alpha' V\partial\psi).$$

The anomaly charge is $c^X = D + 6\alpha' V^2$. OPE of two left-moving X is as usual and is given by

$$X(z)^\mu X(w)^\nu = -\frac{\alpha'}{2}\eta^{\mu\nu}\log(z-w)$$

whereas OPE of several exponents is changed as follows

$$\prod_j e^{2ik_j X(z_j)} = (2\pi)^D \delta\left(\sum_i k_i + iV\right)\prod_{m<n}(z_m - z_n)^{2\alpha' k_m k_n}.$$

$e^{2ikX(z)}$ is still a primary operator but with a weight $\alpha'(k^2 + ikV)$. Also a transformation law for the field X is changed to be

$$f \circ X_\mu(z) = X^\mu(f(z)) + \alpha' V^\mu \log f'.$$

The BRST charge (3) looks as usual in terms of stress tensors and supercurrents but new T^X and G should be used. Nilpotency of Q dictates

$$D + 4\alpha' V^2 - 10 = 0.$$

Now we calculate the cubic action of the fermionic SFT (6) on a string field truncated to weight zero (10) using new CFT. After some tedious algebra one may get (integrating out on the fly field s_μ)

$$S_0 = \frac{1}{g_o^2}\int d^D x e^{-V_\mu x^\mu}\left(\frac{1}{4\alpha'}u^2(x) - \frac{1}{2}\partial_\mu\tau^2(x) + \frac{1}{4\alpha'}\tau^2 + \right.$$
$$\left. + \frac{9}{16}\left(\frac{4}{3\sqrt{3}}\right)^{\alpha' V^2}\tilde{\tau}^2(x)\tilde{u}(x)\right)$$

with

$$\tilde{\varphi}(x) = e^{-\alpha'\left(\log\frac{4}{3\sqrt{3}}\right)\Box}\varphi(x).$$

Analogous computation can be found in [17]. Setting $V = 0$ one restores the usual action coming from SFT. Also we notice that

$$e^{+\alpha'\left(\log\frac{4}{3\sqrt{3}}\right)\Box}e^{-V_\mu x^\mu} = \left(\frac{4}{3\sqrt{3}}\right)^{\alpha' V^2} e^{-V_\mu x^\mu}.$$

3.3. Cosmological Rolling Tachyon in Dilaton Background

Action for the gravity sector reads

$$S_g = \int d^D x\sqrt{-g}\frac{e^{-\Phi}}{2\kappa^2}\left(R + \partial_\mu\Phi\partial^\mu\Phi - g_u U(\Phi)\right). \tag{21}$$

$[g_u] = \text{length}^{-2}$. $U(\Phi)$ is a dilaton potential introduced for generality. If exists, it would be good if the canonical dilaton property that $\Phi+$const results at most in a renormalization of coupling constants is preserved. Moreover this potential may contain non-local operators acting on the dilaton field. In the rest we assume that $U(\Phi)$ does not depend on the metric keeping such a possibility as an open question for the further study. Action motivated by the open string field theory in the linear dilaton background reads

$$S_o = \int d^D x\sqrt{-g}\frac{1}{g_o^2}\left(e^{-\Phi/2}\left(-\frac{1}{2}g^{\mu\nu}\partial_\mu\tau\partial_\nu\tau + \frac{1}{2\alpha'}\tau^2\right) - \frac{1}{\alpha'}\widehat{e^{-\Phi/2}v(\tilde{\tau})}\right). \tag{22}$$

Here hat denotes an operator inverse to the tilde such that

$$\hat{\varphi} = \mathcal{G}_d(\alpha'\Box)\varphi \text{ and } \tilde{\hat{\varphi}} = \varphi,$$

$\mathcal{G}_d$ is supposed to be an analytic function of the argument. The coupling of the dilaton to the open string tachyon potential may be hidden by virtue of the property

$$\int d^D x\sqrt{-g}A\Box B = \int d^D x\sqrt{-g}B\Box A + \text{boundary terms}$$

yielding

$$S_o = \int d^D x \sqrt{-g} \frac{e^{-\Phi/2}}{g_o^2} \left(-\frac{1}{2} g^{\mu\nu} \partial_\mu \tau \partial_\nu \tau + \frac{1}{2\alpha'} \tau^2 - \frac{1}{\alpha'} \widehat{v(\tilde{\tau})} \right). \tag{23}$$

We are interested in a minimal gravitational coupling of lightest open and closed string modes using the action

$$S = S_g + S_o. \tag{24}$$

3.4. Cosmological consequences

Now we turn to an analysis of the cosmological consequences. Scales related to the dilaton should be of cosmological magnitudes. For instance, the present Hubble parameter is related to the Planck mass as $H_{\mathrm{our}} \approx 10^{-60} M_P$. The same level of smallness is applicable to a rate of change of the dilaton field. Indeed, in the dilatonic gravity $\dot{\Phi}$ determines a speed of change of the Newton's constant. Thus $\dot{\Phi}$ should be extremely small. For instance, assuming that Newton's constant changes at most e times during the life-time of the universe we conclude that in a model with a linear dilaton one should have $\Phi \approx \pm H_{\mathrm{our}} t$. It looks counterintuitive that such small quantities can affect somehow processes related to the tachyon condensation. The reason is that all ingredients related to string excitations have α' as a scale. α' is the string length squared and can be written as $1/M_s^2$ where M_s is the string mass. This string mass in any case cannot be less than 1 TeV to be compatible with present experiments. Moreover, M_s is often associated with the Planck mass M_P.

Examining an analog of equation (15) which is now

$$\Box \zeta - g^{\mu\nu} \frac{\partial_\mu \Phi \partial_\nu \zeta}{2} + \frac{\zeta}{\alpha'} - \frac{v''(\tau_0)}{\alpha'} e^{\Phi/2} \left(\widetilde{\widehat{e^{-\Phi/2}} \tilde{\zeta}} \right) = 0. \tag{25}$$

we observe that parameters γ and β introduced above play a crucial role and new interesting phenomena can emerge. Let us make an approximation which produces a very useful toy equation

$$\Box \zeta + \frac{\zeta}{\alpha'} - (1+\epsilon) \frac{v''(\tau_0)}{\alpha'} \tilde{\tilde{\zeta}} = 0. \tag{26}$$

Compared to equation (25) we have dropped a term proportional to the speed of the dilaton[1] and assumed that exponents of the dilaton with all non-local operators acting on them can be accounted as a constant ϵ. This

[1]Strictly speaking this is done to make an analytic analysis possible. Further one can demonstrate numerically that restoring this term does not change the main effect described.

latter constant is positive for non-local operators coming from the SFT. Similar to (17)

$$\omega_\epsilon^2 = \frac{W((1+\epsilon)\gamma)-\beta}{\beta\alpha'}, \quad \gamma = v''(\tau_0)\beta e^\beta \tag{27}$$

and we see that we have shifted the parameter γ and consequently ω^2. For a linear dilaton $\Phi = -2V_0 t$ this shift ϵ can be computed in the SFT to be of order $\alpha' V_0^2$, $[V_0] = \text{length}^{-1}$. Accounting the above discussion about cosmological scales we see that $\epsilon \sim \alpha' H^2$. For these scales a constant H approximation is easily justified and the dynamics of a linearized tachyon $\tau = \tau_0 + \zeta_\epsilon$ is given by an analog of (20)

$$\zeta_\epsilon = e^{-\frac{3}{2}Ht}\mathrm{Re}\left(\zeta_+ e^{\frac{1}{2}t\sqrt{9H^2-4\omega_\epsilon^2}} + \zeta_- e^{-\frac{1}{2}t\sqrt{9H^2-4\omega_\epsilon^2}}\right). \tag{28}$$

Unshifted ω^2 is entirely determined by the open SFT parameters β and γ. It is very difficult to imagine that stringy processes may give as a final state somewhat with scales of the present day cosmology. Corresponding fine-tuning should be of an extreme precision. If ω^2 is finite then most likely it is finite in units of α'. Such an ω^2 dominates under the square root in (20) and oscillations become very frequent, and the suppression factor becomes huge in cosmological scales. A shift of order $\epsilon \sim \alpha' H^2$ will not change anything. This is why we have postponed a discussion on cosmological application before introducing the dilaton.

A distinguished case is $\omega^2 = 0$ which may have a signature in the late cosmology. To have $\omega^2 = 0$ one has to require $v''(\tau_0) = 1$ irrespectively of β. This means that the full tachyon potential including the mass term should have zero second derivative in the minimum[2]. In other words the tachyon should become a long-range field in the true minimum. Parameter γ becomes

$$\gamma = \beta e^\beta, \quad \gamma \geq -\frac{1}{e} \text{ for real } \beta, \quad \gamma = -\frac{1}{e} \text{ for } \beta = -1. \tag{29}$$

In this case of $\omega^2 = 0$ (i.e. $v''(\tau_0) = 1$) the dilaton which provides $\epsilon \sim \alpha' H^2$ can change the behavior drastically. Indeed, the expression $9H^2 - 4\omega_\epsilon^2$ under the square root can become negative. Moreover, if parameter $\beta = -1$ then a positive ϵ makes ω^2 complex at once according to the properties of the Lambert function. This scenario produces oscillations in ζ with a frequency that can be observed at cosmological scales.

Corrections to a constant H emerge and these correction can be computed using the technique elaborated in [1]. One can use the corresponding analysis now with an inclusion of the dilaton field which does not complicate

[2] One can check that for this purpose one has to have at least three terms of a different degree greater than 2 of $\tilde{\tau}$ in v provided v is polynomial. It is natural to expect this kind of a potential from the SFT when more mass levels are included in the calculation of the tachyon potential.

calculations. Here we just briefly sketch what happens with a dynamical tachyon. Once we got an oscillating tachyon these oscillations are translated to the dilaton and the Hubble parameter. Taking a solution for the Hubble parameter and the dilaton and making a linearization around it one will find corrections which obviously will have oscillations (a frequency and a suppression factor will be doubled compared to the tachyon field). Even if H is not a constant the $\dot{H}/H$ ratio is of the same order as $\dot{G}/G$ [18]. Thus it is valid to take a constant H approximation at the beginning. Further, using a relation

$$w = -1 - \frac{2\dot{H}}{3H^2}$$

one finds that the equation of state parameter should be also an oscillating quantity.

To get an insight in numbers we derive let us proceed with a linear dilaton $\Phi = -2V_0 t$. We take $V_0 = \sigma H_{\text{our}}$ meaning that during the evolution of the universe the Newton's constant has changed $e^{2\sigma}$ times, σ is dimensionless. This gives a shift $\epsilon = \sigma^2\alpha' H_{\text{our}}^2 = \sigma^2 10^{-120} M_P^2/M_s^2 = \sigma^2 10^{-120+2n}$ where we put $M_s = 10^{-n} M_P$ with $0 \leq n \leq 16$. The upper bound on n gives strings of TeV mass which is a minimal string mass compatible with current experiments.

Passing from (17) to (27) under our assumption that the full tachyon potential has zero second derivative in the non-perturbative minimum, i.e. $v''(\tau_0) = 1$, two significantly different situations appear: $\gamma > -1/e$ and $\gamma = -1/e$.

The most interesting one is $\gamma > -1/e$, i.e. $\beta \neq -1$ (see (29)). In this case

$$\omega_\epsilon^2 \approx \frac{\epsilon}{(1+\beta)\alpha'}$$

and for the estimated $\epsilon \sim \sigma^2\alpha' H_{\text{our}}^2$ solution (28) becomes

$$\zeta_\epsilon \approx e^{-\frac{3}{2}H_{\text{our}}t}\text{Re}\left(\zeta_+ e^{\frac{1}{2}H_{\text{our}}t\sqrt{9-4\frac{\sigma^2}{1+\beta}}} + \zeta_- e^{-\frac{1}{2}H_{\text{our}}t\sqrt{9-4\frac{\sigma^2}{1+\beta}}}\right).$$

Negative expression under the square root immediately gives oscillations. In the open SFT truncated up to level zero β becomes -0.523. This means that oscillations come out of the latter solution if $\sigma > 1.036$. Taking $\sigma = 1.1$ we get the frequency of oscillations in ζ_ϵ is approximately $H_{\text{our}}/2$. As it was mentioned, this frequency is doubled and is equal to H_{our} in the Hubble parameter and the state parameter. This produces oscillations with a period of order 10 Gyr which is of order of the universe age. Taking other value of β which can result from an inclusion of the higher massive modes in the SFT analysis we may get smaller values of the period of the oscillations. Taking $\beta = -0.95$ and $\sigma = 1.1$ the period of oscillations of the Hubble parameter and the state parameter becomes of order 1 Gyr. The closer β to -1 from above the shorter period is. The higher the dilaton speed the shorter period is as well. Thus we see that in the case $\gamma > -1/e$ one can have an effect of oscillations in the Hubble parameter and the state parameter w accessible for observations.

This number is intriguingly close to the period of oscillations in the z-distribution of quasar spectra observed and confirmed recently [19] (here z is the redshift parameter). The reported value is $(0.15 - 0.65)$ Gyr.

In the case $\gamma = -1/e$ (i.e. $\beta = -1$) one has $\alpha'\omega_\epsilon^2 \approx -i\sqrt{2\epsilon}$, this term most likely dominates under the square root in (28) and the resulting frequency becomes of order $\sqrt{\sigma}10^{+30-n/2}H_{\mathrm{our}}$. In order to make the period of oscillations of order 1 Gyr we have to assume $\sigma \sim 10^{-46}$ for the TeV string mass with $n = 16$. This means that the dilaton changes very slowly, but it is difficult to figure out an appearance of a new small parameter $V_0 \sim 10^{-106}M_P$.

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