Deformed Symmetries in Noncommutative Field Theories*

Marija Dimitrijević†
University of Belgrade, Faculty of Physics,
Studentski trg 12-16, 11000 Beograd, SERBIA

Abstract

In this lecture we discuss the problem of symmetries in noncommutative field theories. The twist approach is used to derive deformed symmetries. In this approach the algebra of symmetry generators remains undeformed, but the Leibniz rules for the generators change. We give some examples. In the case of the canonically deformed space the twisted Poincare symmetry is analyzed and the problem of nonconserved charges is discussed. A way to overcome this problem is to introduce a dynamical noncommutativity and we briefly describe it.

1. Introduction

The concept of a noncommutative (NC) space first appeared when people were discussing divergences in quantum field theory [2]. Our motivation for working with NC spaces comes from the fact that we do not know much about physics at very small scales (or at very high energies). Therefore, we can assume that in this situation coordinates do not commute anymore and we no longer have continuous space-time; more motivation can be found in [3].

On the other hand, the concept of symmetry is very important in physics. Classically, symmetries are described by Lie groups or Lie algebras and the physical space is the representation space of the symmetry algebra. The question arises if one can generalize this notions to noncommutative (deformed) spaces as well. It comes out that there is well defined way to introduce deformed (twisted) symmetries and it is given by the twist formalism, which we describe in this lecture. Using this formalism, in the last section an action invariant under the full classical Poincaré symmetry is constructed and conserved quantities are derived.

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† e-mail address: dmarija@phy.bg.ac.yu

2. Noncommutative spaces, general setting

A noncommutative space $\hat{\mathcal{A}}_{\hat{x}}$ is generated by $\hat{x}^\mu$ coordinates with $\mu = 0, 1, \dots n$, and can be introduced as a quotient

$$\hat{\mathcal{A}}_{\hat{x}} = \frac{\mathbb{C}[\hat{x}^0, \dots, \hat{x}^n][[h]]}{I_{\hat{\mathcal{R}}}}. \tag{1}$$

Here the two-sided ideal $I_{\hat{\mathcal{R}}}$ is given by the linear span of elements

$$I_{\hat{\mathcal{R}}}: \qquad (\hat{x}\dots\hat{x})\Big([\hat{x}^\mu, \hat{x}^\nu] - i\Theta^{\mu\nu}(\hat{x})\Big)(\hat{x}\dots\hat{x}), \tag{2}$$

where $(\hat{x}\dots\hat{x})$ stands for an arbitrary product of the coordinates $\hat{x}^\mu$ in the algebra $\mathbb{C}[\hat{x}^0, \dots, \hat{x}^n][[h]]$. The algebra $\mathbb{C}[\hat{x}^0, \dots, \hat{x}^n][[h]]$ is freely generated by $\hat{x}^\mu$ coordinates and formal power series in the parameter h are included. We also have that $\Theta^{\mu\nu}(\hat{x}) \in \mathbb{C}[\hat{x}^0, \dots, \hat{x}^n][[h]]$ and for $h = 0$ the usual algebra of commuting coordinates is obtained.

The defining relation of the deformed space

$$[\hat{x}^\mu, \hat{x}^\nu] = i\Theta^{\mu\nu}(\hat{x}), \qquad \mu = 0, \dots n, \tag{3}$$

is very general and one usually considers some special examples of it. Among them there are three very important ones

$$\text{Canonically deformed} \quad [\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu}, \tag{4}$$

$$\text{Lie algebra deformed} \quad [\hat{x}^\mu, \hat{x}^\nu] = iC^{\mu\nu}_{\lambda}\hat{x}^\lambda, \tag{5}$$

$$q\text{-deformed} \quad \hat{x}^\mu\hat{x}^\nu = \frac{1}{q}R^{\mu\nu}{}_{\rho\sigma}\hat{x}^\rho\hat{x}^\sigma. \tag{6}$$

In the case of the canonically deformed spaces (which from now on we call θ-deformed space) [4], $\theta^{\mu\nu} = -\theta^{\nu\mu}$ is an antisymmetric constant matrix of mass dimension -2. For Lie algebra deformed spaces [5] $C^{\mu\nu}_{\lambda}$ are Lie algebra structure constants of mass dimension -1. And finally, $R^{\mu\nu}{}_{\rho\sigma}$ is the dimensionless R-matrix of the quantum space [6]. These three examples are important because they fulfill the Poincaré-Birkoff-Witt (PBW) property. This property enables us to map an arbitrary element $\hat{f}(\hat{x})$ of $\hat{\mathcal{A}}_{\hat{x}}$ to the space of commuting coordinates $\mathcal{A}_x$.

To extend the vector space isomorphism to an algebra morphism one has to map the multiplication in the abstract algebra $\hat{\mathcal{A}}_{\hat{x}}$ to the space of commuting coordinates $\mathcal{A}_x$. Let $\hat{f}(\hat{x})$ and $\hat{g}(\hat{x})$ be two elements of $\hat{\mathcal{A}}_{\hat{x}}$. Their product is an element of $\hat{\mathcal{A}}_{\hat{x}}$:

$$\hat{f}(\hat{x})\hat{g}(\hat{x}) = \hat{f}\cdot\hat{g}(\hat{x}) \in \hat{\mathcal{A}}_{\hat{x}} \tag{7}$$

It is mapped to the algebra of commuting variables $\mathcal{A}_x$

$$\hat{f} \cdot \hat{g}(\hat{x}) \mapsto f \star g(x) \in \mathcal{A}_x. \tag{8}$$

Its image is labeled as $f \star g(x)$ and it defines the star product ($\star$-product) of two functions. This product is bilinear and associative but noncommutative. The algebra of noncommuting coordinates $\hat{\mathcal{A}}_{\hat{x}}$ is then isomorphic to the algebra of commuting variables with the $\star$-product (instead of the usual pointwise multiplication) as multiplication, which we label as $\mathcal{A}_x^\star$. The $\star$-product for the θ-deformed space is given by the Moyal $\star$-product [7]

$$\begin{aligned} f \star g\,(x) &= \mu\Big(e^{\frac{i}{2}h\theta^{\rho\sigma}\partial_\rho \otimes \partial_\sigma} f \otimes g\Big) \\ &= \sum_{n=0}^{\infty} \left(\frac{i}{2}\right)^n \frac{1}{n!} \theta^{\rho_1\sigma_1} \ldots \theta^{\rho_n\sigma_n} \Big(\partial_{\rho_1} \ldots \partial_{\rho_n} f(x)\Big)\Big(\partial_{\sigma_1} \ldots \partial_{\sigma_n} g(x)\Big), \end{aligned} \tag{9}$$

with the pointwise multiplication μ

$$\mu(f \otimes g) = f \cdot g. \tag{10}$$

Once again we remind the reader that the deformation parameter h is absorbed in $\theta^{\rho\sigma}$.

Derivatives

Derivatives can be introduced as maps of the noncommutative space $\hat{\mathcal{A}}_{\hat{x}}$ to itself. They are a deformation of the usual derivatives and one can make the following ansatz

$$[\hat{\partial}_\rho, \hat{x}^\mu] = \delta_\rho^\mu + f_\rho^\mu(\hat{\partial}, \theta). \tag{11}$$

Here $f_\rho^\mu(\hat{\partial}, \theta)$ is an operator on the algebra $\hat{\mathcal{A}}_{\hat{x}}$. Furthermore, $[\hat{\partial}_\rho, \hat{\partial}_\sigma] = 0$, that is derivatives commute among themselves.

Since the derivatives $\hat{\partial}_\rho$ are maps of the θ-deformed space into itself the relation (11) has to be consistent with (4)

$$\hat{\partial}_\rho\Big([\hat{x}^\mu, \hat{x}^\nu] - i\theta^{\mu\nu}\Big) = \Big([\hat{x}^\mu, \hat{x}^\nu] - i\theta^{\mu\nu}\Big)\hat{\partial}_\rho = 0. \tag{12}$$

In our example of the θ-deformed space we obtain

$$[\hat{\partial}_\rho, \hat{x}^\mu] = \delta_\rho^\mu. \tag{13}$$

There are no additional terms in (13). This is due to the fact that the right hand side of (4) is constant. The representation of $\hat{\partial}_\rho$ on $\mathcal{A}_x^\star$ is given by the usual partial derivative

$$\hat{\partial}_\rho \mapsto \partial_\rho^\star = \partial_\rho. \tag{14}$$

This derivative has the undeformed Leibniz rule

$$\begin{aligned}(\partial^\star_\rho(f\star g)) = \partial_\rho(f\star g) &= (\partial^\star_\rho f)\star g + f\star(\partial^\star_\rho g)\\ &= (\partial_\rho f)\star g + f\star(\partial_\rho g).\end{aligned} \tag{15}$$

Integral

To be able to write down actions we have to introduce an integral on this space. One can check that the usual integral on the commutative space is cyclic[1], that is it fulfills

$$\int \mathrm{d}^4x\ f\star g = \int \mathrm{d}^4x\ g\star f = \int \mathrm{d}^4x\ f\cdot g. \tag{16}$$

Note that from now on we work in 4 dimensions. However all the results in this chapter can be generalized to higher dimensions as well. From (16) it follows

$$\int \mathrm{d}^4x\ (f_1\star f_2\star\cdots\star f_k) = \int \mathrm{d}^4x\ (f_k\star f_1\star\cdots\star f_{k-1}), \tag{17}$$

that is cyclic permutations under the integral are allowed. This is important if we want to define the variational principle which is used to derive equations of motion.

3. Twist formalism

There is a mathematically well defined way to deform classical[2] symmetries. It is given by the twist procedure, which was first introduced by Drinfel'd in [8]. Let us briefly motivate the introduction of this formalism. First we notice that the $\star$-product (9) can be viewed as coming from an action of a bilinear differential operator (called Abelian twist) which is given by

$$\mathcal{F} = e^{-\frac{i}{2}\theta^{\rho\sigma}\partial_\rho\otimes\partial_\sigma} \tag{18}$$

as

$$\begin{aligned}f\star g = \mu\{\mathcal{F}^{-1} f\otimes g\} &= \mu\{e^{\frac{i}{2}\theta^{\rho\sigma}\partial_\rho\otimes\partial_\sigma} f\otimes g\}\\ &= f\cdot g + \frac{i}{2}\theta^{\rho\sigma}(\partial_\rho f)\cdot(\partial_\sigma g) + \mathcal{O}(\theta^2).\end{aligned} \tag{19}$$

Here μ is the usual pointwise multiplication $\mu(f\otimes g) = f\cdot g$.

[1] Note that (16) is fulfilled if suitable boundary conditions at infinity are chosen.

[2] Note that in this contribution "classical", "ordinary" and "undeformed" symmetry/theory refers to the symmetry/theory on the commutative space.

From (4) it is obvious that this deformation breaks the classical Lorentz symmetry. Namely, $\theta^{\mu\nu}$ on the right-hand side of (4) is an arbitrary constant matrix, so it does not transform under Lorentz transformations, while the left-hand side transforms. However, one can construct a deformation of Lorentz symmetry such that it is a symmetry of (4). The way to do this construction is to use the twist formalism.

The ingredients we need are:

- a Lie algebra g, generated by N elements t^a. For example, the algebra of undeformed Lorentz or Poincaré transformations.
- an action of the Lie algebra on the space one wants to deform.
- a twist element $\mathcal{F} \in Ug \otimes Ug$, where Ug is the universal enveloping algebra of g. Ug is a Hopf algebra [9], in particular there is a linear map, called coproduct

  $$\Delta : Ug \longrightarrow Ug \otimes Ug, \qquad \Delta(t^a) = t^a \otimes 1 + 1 \otimes t^a. \tag{20}$$

  From the coproduct the Leibniz rule for the generators follows. The main property $\mathcal{F}$ has to satisfy is the cocycle condition

  $$\mathcal{F} \otimes 1(\Delta \otimes id)\mathcal{F} = 1 \otimes \mathcal{F}(id \otimes \Delta)\mathcal{F} \tag{21}$$

  and it ensures the associativity of the $\star$-product (19). Additionally we require that $\mathcal{F} = 1 \otimes 1 + \mathcal{O}(h)$, with the deformation parameter h. This on the other hand ensures that in the commutative limit $h \to 0$ the deformation disappears.

The twist $\mathcal{F}$ acts on the universal enveloping algebra (Hopf algebra) Ug and gives as a result a twisted Hopf algebra $Ug_{\mathcal{F}}$. The algebra relations do not change, but the comultiplication (20) is deformed

$$\Delta_{\mathcal{F}}(t^a) = \mathcal{F}\Delta(t^a)\mathcal{F}^{-1}, \tag{22}$$

leading to the deformed Leibniz rules for the symmetry generators. Additionally, the twist $\mathcal{F}$ acts on the space of functions of commuting coordinates $\mathcal{A}_x$ introducing a new product, called $\star$-product

$$\begin{aligned} \mu_\star(f \otimes g) &\equiv \mu \circ \mathcal{F}^{-1}(f \otimes g) \\ &= (\bar{\mathrm{f}}^\alpha f)(\bar{\mathrm{f}}_\alpha g) = f \star g. \end{aligned} \tag{23}$$

Here we introduced the notation $\mathcal{F} = \mathrm{f}^\alpha \otimes \mathrm{f}_\alpha$ and $\mathcal{F}^{-1} = \bar{\mathrm{f}}^\alpha \otimes \bar{\mathrm{f}}_\alpha$. The $\star$-product (23) is associative (provided that $\mathcal{F}$ fulfills the cocycle condition (21)) but noncommutative and it is a deformation of the usual pointwise multiplication.

The twist formalism is used in [10] to deform commutative gauge and gravity theories and also supersymmetry. For simplicity we here take as an example the θ-deformed Poincaré symmetry. The transformation law of a scalar field under the deformed Lorentz transformations is defined as follows

$$\delta^\star_\omega \phi(x) = -\omega^\lambda{}_\nu x^\nu (\partial_\lambda \phi) = -X^\star_\omega \star \phi(x), \tag{24}$$

$$= -\omega^\lambda{}_\nu x^\nu \star (\partial_\lambda \phi) - \frac{i}{2}\theta^{\rho\sigma}\omega^\lambda{}_\rho (\partial_\sigma \partial_\lambda \phi). \tag{25}$$

The twist $\mathcal{F} = e^{-\frac{i}{2}\theta^{\rho\sigma}\partial_\rho \otimes \partial_\sigma}$ composed of the generators of translations ∂_ρ introduces the Moyal-Weyl $\star$-product (9) via (23). It also acts on the Poincaré Hopf algebra, twisting the coproduct of the generators. This leads to the deformed Leibniz rule for the Lorentz transformations (24). In the case of the $\star$-product of two scalar fields we obtain

$$\begin{aligned} \delta^\star_\omega \Big(\phi_1 \star \phi_2\Big) &= (\delta^\star_\omega \phi_1) \star \phi_2 + \phi_1 \star (\delta^\star_\omega \phi_2) \\ &\quad - \frac{i}{2}\theta^{\rho\sigma}\Big((\delta^\star_{\partial_\rho \omega}\phi_1) \star (\partial_\sigma \phi_2) + (\partial_\rho \phi_1) \star (\delta^\star_{\partial_\sigma \omega}\phi_2)\Big) \\ &= -\omega^\lambda{}_\nu x^\nu \partial_\lambda (\phi_1 \star \phi_2). \end{aligned} \tag{26}$$

That is, the $\star$-product of two scalar fields is a scalar field again.

Using this results one can construct actions invariant under the θ-deformed Poincaré symmetry. For example, the action for ϕ^4 scalar field theory

$$S = \int \mathrm{d}^4 x \; \{\frac{1}{2}(\partial_\mu \phi) \star (\partial^\mu \phi) - \frac{m^2}{2}\phi \star \phi - \frac{\lambda}{4!}\phi \star \phi \star \phi \star \phi\} \tag{27}$$

is invariant under θ-deformed Poincaré symmetry provided that one uses the deformed Leibniz rule (26). Using this invariance the energy-momentum tensor $T^\mu_{\ \rho}$ can be constructed. It is conserved but it is not symmetric $T^{\mu\rho} \neq T^{\rho\mu}$. However, the explicitly calculated angular momentum $M^{\alpha\beta\mu}$ is not conserved on the equations of motion [11]. In general, the Noether construction of conserved quantities which works very well in the commutative field theory cannot be applied in the case of deformed symmetries. A generalization of this method is needed and that is subject of the ongoing research. A way to overcome this problem is to introduce the dynamical noncommutativity and we present it in the next section.

4. Dynamical noncommutativity

The Moyal-Weyl $\star$-product (9) can be generalized as

$$f \star g = \mu\{\mathcal{F}^{-1} f \otimes g\} = \mu\{e^{\frac{i}{2}\theta^{ab}X_a \otimes X_b} f \otimes g\}, \tag{28}$$

where θ^{ab} is a constant antisymmetric matrix. The twist $\mathcal{F}$ is given by

$$\mathcal{F} = e^{-\frac{i}{2}\theta^{ab}X_a \otimes X_b} \tag{29}$$

and $X_a = e_a{}^\mu(x)\partial_\mu$ are D commuting vector fields, the index a being just a label for the vector fields. The coordinates x^μ span a D-dimensional Minkowski space with metric $\eta_{\mu\nu}$. In the commutative limit ($\theta^{ab} \to 0$) the product (18) reduces to the usual pointwise multiplication. The requirement that the vector fields X_a commute ensures the associativity of (18). From $[X_a, X_b] = 0$ and supposing that the square matrix $e_a{}^\mu$ has an inverse $e_\mu{}^a$ everywhere, we obtain the condition $\partial_{[\mu} e_{\nu}{}^{a]} = 0$. It is solved by

$$e_\nu{}^a(x) = \partial_\nu \Phi^a(x). \tag{30}$$

In this way the $\star$-product (18) is determined by D scalar fields subject to the condition that $\partial_\nu \Phi^a$ is everywhere invertible. The $\star$-product (28) is invariant under Lorentz transformations, as it follows from the Lorentz invariance of the vector fields $X_a = e_a{}^\mu(x)\partial_\mu$. Since X^a commute, the action of X_a satisfies the Leibniz rule:

$$X_a(f \star g) = (X_a f) \star g + f \star (X_a g) \tag{31}$$

whereas a deformed Leibniz rule holds for the usual partial derivatives ∂_μ [10].

Action

We use the $\star$-product (28) to define an action for $\Phi^{\star 4}$ theory coupled to Φ^c. But first we have to define a cyclic integral. Namely, using the $\star$-product (28) we have

$$\begin{aligned}\int \mathrm{d}^4x \; f \star g &= \int \mathrm{d}^4x \, \Big(g \star f + X_a\big(\frac{\sinh \Delta}{\Delta}(f, \tilde{X}^a g)\big)\Big)\\ &\neq \int \mathrm{d}^4x \, \Big(f \star g + \text{ boundary terms}\Big),\end{aligned}$$

with $\Delta(f,g) \equiv \frac{i}{2}\theta^{ab}(X_a f)(X_b g)$. The cyclicity is restored introducing a measure function $e \equiv \det e_\mu{}^a$ and one can check that the integral $\int \mathrm{d}^4x \; e(f \star g)$ fulfills (16).

$$\begin{aligned}S[\Phi, \Phi^a] = \int \mathrm{d}^4x \, \Big[\Big(\frac{1}{2}\partial_\mu \Phi \star \partial^\mu \Phi - \frac{m^2}{2}\Phi \star \Phi - \frac{\lambda}{4!}\Phi \star \Phi \star \Phi \star \Phi \\ + \frac{1}{2}\partial_\mu \Phi^c \star \partial^\mu \Phi_c\Big) \star e^{-1}\Big] e \, .\end{aligned} \tag{32}$$

In the commutative limit this action reduces to the commutative Φ^4 action for field Φ and the free action for massless scalar fields Φ^c. Note that the fields Φ and Φ^c are decoupled in the commutative limit.

Conserved quantities

The action (32) is invariant under global translations:

$$x^\mu \to x'^\mu = x^\mu + \epsilon^\mu, \qquad \delta_\epsilon \Phi = -\epsilon^\mu \partial_\mu \Phi, \qquad \delta_\epsilon \Phi^a = -\epsilon^\mu \partial_\mu \Phi^a.$$

Varying the action (32) leads to

$$\begin{aligned} 0 = \delta S = \int \mathrm{d}^4 x\, \Big(& \delta_\Phi [(\mathcal{L}_\star \star e^{-1})e] + \delta_{\Phi^c}[(\mathcal{L}_\star \star e^{-1})e] \\ & + \epsilon^\mu \partial_\mu [(\mathcal{L}_\star \star e^{-1})e] + (\mathcal{L}_\star \star e^{-1}) e (\partial_\mu \epsilon^\mu)\Big) \\ = \int \mathrm{d}^4 x\, & (\epsilon^\rho (\partial_\mu T^\mu{}_\rho)), \end{aligned} \tag{33}$$

where

$$T^\mu_\nu = -\frac{e}{2}(\partial_\nu \Phi)\{\partial^\mu \Phi \overset{\star}{,} e^{-1}\} - \frac{e}{2}(\partial_\nu \Phi_c)\{\partial^\mu \Phi^c \overset{\star}{,} e^{-1}\} \tag{34}$$

$$+ e e_a{}^\mu \Big(\mathcal{L}_\star \star (e^{-1}\partial_\nu \Phi^a) - \frac{e^(\Delta) - 1}{\Delta}(X_c \mathcal{L}_\star, \tilde{X}^a(e^{-1}\partial_\nu \Phi^c))\Big) \tag{35}$$

is the conserved energy-momentum tensor. This tensor is not symmetric: only in the commutative limit it reduces to the canonical (and symmetric) energy-momentum tensor of the decoupled Φ and Φ^c fields.

The action (32) is also invariant under global Lorentz rotations:

$$x^\mu \to x'^\mu = x^\mu + \omega^\mu{}_\rho x^\rho, \quad \delta_\epsilon \Phi = -\omega^\mu{}_\rho x^\rho (\partial_\mu \Phi), \quad \delta_\epsilon \Phi^a = -\omega^\mu{}_\rho x^\rho (\partial_\mu \Phi^a).$$

Repeating the same procedure as in the case of translations we obtain the conserved (on the equations of motion) angular momentum tensor

$$\begin{aligned} M^\mu{}_{\nu\rho} = & \frac{e}{2} x_{[\nu}\partial_{\rho]}\Phi \{\partial^\mu \Phi \overset{\star}{,} e^{-1}\} + \frac{e}{2} x_{[\nu}\partial_{\rho]}\Phi_c \{\partial^\mu \Phi^c \overset{\star}{,} e^{-1}\} \\ & - e e_a{}^\mu \Big(\mathcal{L}_\star \star (e^{-1} x_{[\nu}\partial_{\rho]}\Phi^a)\Big) + e e_a{}^\mu \Big[T(\Delta)(X_c \mathcal{L}_\star, \tilde{X}^a(e^{-1} x_{[\nu}\partial_{\rho]}\Phi^c)) \\ & - T(\Delta)\Big(\partial_{[\nu}\Phi^d, \frac{1}{2}\tilde{X}^a(\{\partial_{\rho]}\Phi_d \overset{\star}{,} e^{-1}\})\Big) \\ & - T(\Delta)\Big(\partial_{[\nu}\Phi, \frac{1}{2}\tilde{X}^a(\{\partial_{\rho]}\Phi \overset{\star}{,} e^{-1}\})\Big) \\ & + S(\Delta)(\partial_{[\nu}\Phi, \tilde{X}^a(\partial_{\rho]}\Phi \star e^{-1})) + S(\Delta)(\partial_{[\nu}\Phi_d, \tilde{X}^a(\partial_{\rho]}\Phi^d \star e^{-1}))\Big]. \end{aligned} \tag{36}$$

5. Conclusions

In this lecture we analyzed the problem of finding conserved quantities when the theory is invariant under deformed (twisted) symmetry. There is still no definite solution to this problem and it is a subject of the ongoing research.

However, we managed to overcome the problem of broken Lorentz invariance by constructing a $\star$-product which is invariant under the undeformed Lorentz transformations. At the same time, we implemented a dynamical noncommutativity in $\Phi^{\star 4}$ theory. The field equations of the resulting twisted $\Phi^{\star 4}$ theory admit the vacuum solution $\Phi = 0$, $e_\mu{}^a \equiv \partial_\mu \Phi^a = \delta_\mu^a$. In particular when $e_\mu{}^a = \delta_\mu^a$ the $\star$-product between any two functions reduces to the Moyal-Weyl (9) $\star$-product. Since the theory is invariant under the undeformed Lorentz transformations, we obtained the conserved energy-momentum tensor (34) and conserved angular momentum (36).

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